THOMSON'S NEW SERIES OF MATHEMATICS.

KEY

TO NEW

PRACTICAL ALGEBRA,

FOR TEACHERS.

BY

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NOTE.

ALL agree that the best kind of help for pupils in Arithmetic and Algebra, is *Self-help;* that it is better for the learner not to know the answer to a problem, until he has tried his own ability to solve it. In a word, that it is better for him to solve a single example independently, than a score by the help of a teacher or a Key.

And yet it must be admitted that a majority of teachers desire a KEY. This demand comes not only from young and inexperienced teachers, but from those whose character and scholarship are above suspicion. They desire it, not because of their inability to solve the problems, nor because they shrink from labor. Their object is to save time, which they may devote to other branches of study.

A well constructed Key will often disclose in a single minute the error in a pupil's work, which might consume half an hour of the teacher's time, if he were obliged to wade through a long operation.

The plan of the work before us is to indicate in full the operations to be performed, and give the results; omitting the minor details. It also contains many valuable suggestions as to the different methods by which certain problems may be solved. It is hoped teachers will find it adapted to their wants.

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KEY.

EXERCISES IN NOTATION.

Pages 12, 13.

2. $4c + d + m - 5x = ab.$

3. $5cd + \frac{a}{b} = xy.$

4. $\frac{3b}{5c} + 4m = c + 6d - 7ax.$

5. $a - b + xy = 6mn.$

6. $x - y + 4a + b - m = cd + 15m.$

2. The quotient of twice the product of a and b divided by x, plus a minus b, equals the quotient of a plus b divided by c, plus the product of a, x, and y minus four times the product of c and d.

3. The quotient of three times b plus c divided by 8, plus 3 times x, equals 3 times the product of c and d divided by a, plus the product of x, y, and z minus the quotient of c divided by d.

4. The quotient of 3 times a divided by 5, minus the product of a and x plus the product of b and c, equals the quotient of 4 times a minus b divided by x, plus the quotient of c times d divided by 4, minus 3 times x.

5. The product of a, b, and c minus x, divided by 3 times d, increased or diminished by 3 times x plus 5 times y, equals the product of c, d, and h plus x divided by twice a, minus the product of x and y.

6. The quotient of 4 times a into x into y divided by 5 times a, plus the quotient of a minus b divided by x, equals the quotient of x plus y divided by a, minus the quotient of twice a plus d divided by 3 times c.

ALGEBRAIC OPERATIONS.

Page 15.

3. Let x = price of the apple,
Then will $3x$ = " " " orange.
And $4x$ = 8 cents.
Dividing by 4, x = 2 cents, apple;
And $3x$ = 6 cents, orange. } *Ans.*

4. Let x = value of the hay,
Then will $4x$ = " " " cow.
And $5x$ = $40.
Dividing by 5, x = $8, hay;
And $4x$ = $32, cow. } *Ans.*

5. Let x = one of the numbers,
Then will $3x$ = other number.
And $4x$ = 36.
Dividing by 4, x = 9;
And $3x$ = 27. } *Ans.*

6. Let x = C's number of peaches,
Then $2x$ = B's " " "
And $4x$ = A's " " "
Adding, $7x$ = 28.
Dividing by 7, x = 4, C's number;
And $2x$ = 8, B's "
" $4x$ = 16, A's " } *Ans.*

7. Let x = son's age,
Then will $3x$ = father's age.
And $4x$ = 48.
Dividing by 4, x = 12 years, son's age;
And $3x$ = 36 " father's age. } *Ans.*

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8. Let x = B's share of gain,
Then will $4x$ = A's " "
And $5x$ = \$100.
Dividing by 5, x = \$20, B's share of gain;
And $4x$ = \$80, A's " " } *Ans.*

9. Let x = 1st number,
Then will $2x$ = 2d "
And $3x$ = 3d "
Adding, $6x$ = 90.
Dividing by 6, x = 15, 1st;
And $2x$ = 30, 2d;
" $3x$ = 45, 3d. } *Ans.*

10. Let x = value of the calf,
Then will $8x$ = " " " cow.
And $9x$ = \$63.
Dividing by 9, x = \$7, price of calf;
And $8x$ = \$56, " " cow. } *Ans.*

11. Let x = value of bridle,
Then will $2x$ = " " saddle,
And $21x$ = " " horse.
Adding $24x$ = \$126.
Divid. by 24, x = \$5.25, worth of bridle;
And $2x$ = \$10.50, " " saddle;
" $21x$ = \$110.25, " " horse. } *Ans.*

12. Let x = daughter's portion,
Then $2x$ = son's "
And $9x$ = wife's "
" $12x$ = \$36000.
Therefore, x = \$3000, daughter's portion;
And $2x$ = \$6000, son's "
" $9x$ = \$27000, wife's " } *Ans.*

13. Let $x = 1$st number,
Then $3x = 2$d "
And $4x + 5 = 3$d "
" $8x + 5 = 1877$.
Subt. 5 from each, $8x = 1872$.
Therefore, $x = 234$, 1st number;
And $3x = 702$, 2d "
" $4x + 5 = 941$, 3d "
} *Ans.*

POWERS AND ROOTS.

Page 17.

1–12. Oral.
13. $a^2 + b^2$.
14. $(a + b)^2$.
15. $a + b - c^2$.
16. $\sqrt{a} + \sqrt{x}$.
17. $\sqrt[3]{x} - y^5$.
18. $\sqrt[3]{a} + b^2$.

ALGEBRAIC EXPRESSIONS.

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4. $(a + b)cd - \frac{x}{c} = 5 \times 20 - \frac{6}{4} = 98\frac{1}{2}$.
5. $(x - a) + ax + \frac{c}{a} = 4 + 12 + 2 = 18$.
6. $x \div 2 + (d - c) + bc - x = 3 + 1 + 12 - 6 = 10$.
7. $dx + (c - a)(a - b) + x = 30 - 2 + 6 = 34$.
8. $d + x(c - a) + a - x + c = 5 + 12 + 2 - 6 + 4 = 17$.

FORCE OF THE SIGNS.

Page 21.

1. 60.
2. 40.
3. $ac + 8b$.
4. $5b - 2d$.
5. 35.
6. 24.
7. $3x + 2y + ab$.
8. $6b - 7cx + 3a$.
9. $bxy + cxy$.

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10. $\frac{15xy}{2z} + a$.
11. $\frac{b - a}{xy} + 2z$.

12. $3x + xy + 2z \times 3y = 3x + xy + 6yz$, *Ans.*

13. $(x - y) \times \frac{a - b}{d} = \frac{ax - ay - bx + by}{d}$, *Ans.*

$a = 3$, $b = 4$, $c = 2$, $x = 6$, $y = 8$, and $z = 10$.

14. $a + \frac{ax}{c} + yz = 3 + \frac{18}{2} + 80 = 92$, *Ans.*

15. $\frac{2b}{x - b} + aby + 2z = \frac{8}{2} + 96 + 20 = 120$, *Ans.*

ADDITION.

Case I, Page 24.

3. $21ab$.
4. $17xy$.
5. $15a^2$.
6. $-23bcd$.
7. $-16x^3y^2$.
8. $45ab^2$.
9. $-39abx^2y^2$.
10. $29b^3dm^3$.
12. $bc = 4$.
13. $xy = 5$.

Case II, Page 25.

16. $8x$.
17. abc.
18. $-12b$.
19. $-12y$.
20. $-2m$.

21. $6ab + 14ab + 15ab + 16ab = 51ab$;
$-7ab - 12ab = -19ab$;
$51ab - 19ab = 32ab$.
Since $32ab = 32$, $\therefore ab = 1$, *Ans.*

22. $bcd - 3bcd + 4bcd + 4bcd - 5bcd = 75$.
Uniting, $9bcd - 8bcd = bcd = 75$, *Ans.*

EXAMPLES.

Page 26.

1. $24a + 2b - 3d$.
2. $16mn - xy + bc$.
3. $3bc$
$-7bc + xy - mn$
$11bc$
$9bc$
$16bc + xy - mn$, *Ans.*

4. $5ab - 3mn$
$-ab$
$3ab + 2z$
$-4ab$
ab
$4ab - 3mn + 2z$, *Ans.*

5. $3xy$
$-\ xy + ab$
$-\ 7xy \qquad + b$
$8xy$
$-\ xy$
$13xy$
$15xy + ab + b$, *Ans.*

7. $21\,(a + b)$.

8. $19c\,(x - y)$.

9. $7a\sqrt{xy}$.

10. $6\sqrt{a}$.

11. $10\sqrt{x - y}$.

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13. $a(7 - 6b + 3d - 3m)$.

14. $(ab + 3 - 2c - 5m)\,y$.

15. $m(9 + ab - 7c + 3d)$.

16. $x(13a - 3b + c - 3d + m)$.

17. $(a + b - c)\,xy$.

PROBLEMS.

Page 28.

2. Let $x =$ cost of ball,
Then $2x - 2 =$ " " kite.
And $3x - 2 = 46$ cents.
Adding 2 to each side, $3x = 48$ "
$\therefore\ x = 16$ " ball;
And $2x - 2 = 30$ " kite, } *Ans.*

NOTE.—As the learner is not supposed to be acquainted with transposition, he should in the operation set down the number required to be added to each side of the equation, as seen in the solution of the first example.

3. Let $x =$ number of peaches,
Then $2x - 3 =$ " " pears.
And $3x - 3 = 75$.
Adding 3 to each side, $3x = 78$.
$\therefore\ x = 26$ peaches;
And $2x - 3 = 49$ pears, } *Ans.*

4. Let $x =$ the less number;
Then $5x - 5 =$ the greater number.
And $6x - 5 = 85$.
Adding 5 to each side, $6x = 90$.
$\therefore\ x = 15$, the less No;
And $5x - 5 = 70$, the greater No. } *Ans.*

5. Let x = number of boys,
Then $2x - 5$ = " " girls.
And $3x - 5 = 40$.
Adding 5 to each side, $3x = 45$.
$\therefore \quad x = 15$ boys;
And $2x - 5 = 25$ girls, } *Ans.*

6. $44x + 65x - 24 = 85$
$44x + 65x = 109$
Uniting, $109x = 109$
$\therefore \quad x = 1$, *Ans.*

7. $7x + 2x - 3 = 60$
Uniting, $9x = 63$
$\therefore \quad x = 7$, *Ans.*

8. $4y + 2y + 5y - 7 = 70$
Uniting, $11y = 77$
$\therefore \quad y = 7$, *Ans.*

9. Let x = B's votes,
Then $4x - 20$ = A's "
And $5x - 20 = 450$.
Adding 20 to each side, $5x = 470$.
$\therefore \quad x = 94$, B;
And $4x - 20 = 356$, A, } *Ans*

10. Let x = the less number.
Then $4x - 3$ = the greater number.
And $5x - 3 = 177$.
Adding 3 to each side, $5x = 180$.
$\therefore \quad x = 36$;
And $4x - 3 = 141$, } *Ans.*

11. $4y + 3y + 2y - 12 = 60$
Uniting, $9y = 72$
$\therefore \quad y = 8$, *Ans.*

12. Let x = price of top,
Then $3x - 4$ = " " ball.
And $4x - 4 = 32$ cents.
Adding 4 to each side, $4x = 36$ "
$\therefore \quad x = 9$ cents; } *Ans.*
And $3x - 4 = 23$ " }

13. Let x = price of bridle,
Then $4x - 5$ = " " saddle.
And $5x - 5 = \$40$.
Adding 5 to each side, $5x = \$45$.
$\therefore \quad x = \$9$, bridle; } *Ans.*
And $4x - 5 = \$31$, saddle, }

14. Let x = sum spent in A.M.,
Then $3x - 4$ = " " " P.M.
$4x - 4 = 100$ cents.
Adding 4 to each side, $4x = 104$ "
$\therefore \quad x = 26$c., A.M.; } *Ans.*
And $3x - 4 = 74$c., P.M., }

16. $2x + 5x + 3x - 10 = 130$
Uniting, $10x = 140$
$\therefore \quad x = 14$, *Ans.*
PROOF. $28 + 70 + 42 - 10 = 130$

17. $4x + 3x + 7x - 12 = 86$
Uniting, $14x = 98$
$\therefore \quad x = 7$, *Ans.*
PROOF. $28 + 21 + 49 - 12 = 86$

18. $10x - 4x + 9x - 25 = 155$; $\therefore \quad x = 12$, *Ans.*
PROOF. $120 - 48 + 108 - 25 = 155$

19. $15x - 7x - 2x - 60 = 300$; $\therefore \quad x = 60$, *Ans.*
PROOF. $900 - 420 - 120 - 60 = 300$

20. $18x - 4x + x - 75 = 225$; $\therefore \quad x = 20$, *Ans.*
PROOF. $360 - 80 + 20 - 75 = 225$

SUBTRACTION.

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3. $14xyz$.
4. $-62ab$.
5. $19ab$.
6. $27xy$.
7. $43ac$.
8. $37ax^2$.
9. $51a^2b$.
10. $-44x^2y^2$.

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11. $38a^2b$.
12. 0.
13. $-77m^2x$.
14. $53x^3y$.

15. A debt is properly regarded as a negative or minus quantity. Hence we have to take $-$\$50 from \$100.
$\$100 - (-\$50) = \$100 + \$50 = \$150$, *Ans.*

16. 15°
-10°
25°, *Ans.*

17. \$275
$-$\$145
\$420, *Ans.*

18. $4xy - 6a$.
19. $13b^2 + 16am$.
20. $18x^2 + y^2 + 6a$.
21. $13ab + d - x - 5m + 3n$.
22. $9cd - ab - 2m + 3n + 4y$.
23. $18m - 23$.
24. $12x^2 - 13x$.
25. $16ab + 13c + d$.
26. $a - b + c$.

PROOF.—The difference plus the subtrahend equals the minuend.

$a - b + c$ = difference.
$b - c$ = subtrahend.
a = minuend.

27. $6(a + b)$.
28. $9(a - b + x)$.
29. $5(a + b)$.
30. $-7(x^2 - y)$.

31. \$1000 = the gain.
\$1000 $-$ \$500 $-$ \$100 = \$400, B's share.
Then \$500 + \$100 = \$600, A's " *Ans.*

32. If distances east be regarded as $+$, then those reckoned west must be considered as $-$.

East longitude, 23°
West " -37°
60°, *Ans.*

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34. $(2b - c + d)\,x^2$.
35. $(ab - c - d + x)\,y$.
36. $a^2(7 - b + c)$.
37. $(ab - 3c - d - m)\,x$.
38. $(8 - ab + c - d)\,xy$.
39. $5ac + bmc$
$3ac - dc$
$2ac + bmc + dc$
$(2a + bm + d)\,c$, *Ans.*

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4. $b - (c - d + m) = b - c + d - m$, *Ans.*
5. $5x + y - ab + 4d$.
6. $2a - [b + c - (x + y) - d] =$
$2a - (b + c - x - y - d) =$
$2a - b - c + x + y + d$, *Ans.*
7. $a - b + c - a + c + c - a + b$.

MULTIPLICATION.

Case I, Page 36.

5. $42abc$.
6. $35abcxy$.
7. $8dmxy$.
8. $63bcdxyz$.
9. $56abxy$.
10. $42acdz$.
11. $54bcdm$.
12. $63adfxyz$.

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13. Given.
14. $-45abxy$.
15. $42abcd$.
16. $152abcxy$.
17. $-414abcxy$.
18. $945bcdxy$.

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22. $15x^3y^2$.
23. $24a^3b^3$.
24. $a^6x^5y^2$.
25. a^2b^{m+n}.
26. $6x^2y^2z$.
27. $18a^5b^4c^2$.
28. $3a = 9$;
$a^3 = 27$;
$27 - 9 = 18$, *Ans.*
29. $4x = 16$;
$x^4 = 4^4 = 256$;
$256 - 16 =$
240, *Ans.*
30. $6x^2y$.
31. $-18a^4b^3c$.

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32. $4x^3y^2$.
33. $21a^3b^3$.
34. $40c^4x^ny$.
35. $-28a^4b^3$.
36. $-6x^2y^3$.
37. $21a^2b^2c^4$.
38. $-28a^3c^4$.
39. $x^2y^2z^3$.

Case II, Page 39.

3. $6acx^2 + 8c^2d$.
4. $15a^2b^2x - 6acdx$
$+ 3ax^2$.
5. $-8a^2bd + 6ab^2d$
$- 2bdm^2$.
6. $-15a^4c$
$+ 20a^2b^2c + 10a^2c^3$.

Case III, Page 40.

1. $2a + b$
$3x + y$
$6ax + 3bx + 2ay + by$, *Ans.*

2. $3x + 4y$
$a - b$
$3ax + 4ay - 3bx - 4by$, *Ans.*

3. $4b - c$
$3d - a$
$12bd - 3cd - 4ab + ac$, *Ans.*

4. $6xy - 2a$
$b + c$
$6bxy - 2ab + 6cxy - 2ac$, *Ans.*

5. $3a + 4b - c$
$x - y$
$3ax + 4bx - cx - 3ay - 4by + cy$, *Ans.*

6. $5x + 3y + z$
$a + b$
$5ax + 3ay + az + 5bx + 3by + bz$, *Ans.*

7. $7cdx - 3ab$
$2m - 3n$
$14cdmx - 6abm - 21cdnx + 9abn$, *Ans.*

8. $8abc + 4m$
$3x - 4y$
$24abcx + 12mx - 32abcy - 16my$, *Ans.*

9, 10. Given.

11. $3abc^n xyz^m$.

12. $11abc^2d^{m+n}$.

13. a^2x^{2+n}.

14. $cx(a + b)^4$.

15. $5c(a - b)^5$.

16. $abc(x + y)^{m+n}$.

17. $-3x(a + b)^4$.

Case III—Continued. **Page 41.**

20. $a^2 - ab + b^2$
$a + b$
$a^3 - a^2b + ab^2$
$a^2b - ab^2 + b^3$
$a^3 + b^3$, *Ans.*

21. $a^2 - ab + b^2$
$a^2 + ab + b^2$
$a^4 - a^3b + a^2b^2$
$a^3b - a^2b^2 + ab^3$
$a^2b^2 - ab^3 + b^4$
$a^4 + a^2b^2 + b^4$, *Ans.*

22. $x^2 + x + 1$
$x^2 - x + 1$
$x^4 + x^3 + x^2$
$- x^3 - x^2 - x$
$+ x^2 + x + 1$
$x^4 + x^2 + 1$, *Ans.*

23. $3x^2 - 2xy + 5$
$x^2 + 2xy - 6$
$3x^4 - 2x^3y + 5x^2$
$6x^3y - 4x^2y^2 + 10xy$
$- 18x^2 + 12xy - 30$
$3x^4 + 4x^3y - 13x^2 - 4x^2y^2 + 22xy - 30$, *Ans.*

24. $4ax - 2ay$
$6ax + 3ay$
$24a^2x^2 - 12a^2xy$
$12a^2xy - 6a^2y^2$
$24a^2x^2 - 6a^2y^2$, *Ans.*

25. $d + bx$
$d + cx$
$d^2 + bdx + cdx + bcx^2$, *Ans.*

27. $x + y$
$x - y.$
$x^2 + xy$
$- xy - y^2$
$x^2 - y^2$, *Ans.*

28. $a^3 + 1$
$a + 1$
$a^4 + a$
$a^3 + 1$
$a^4 + a^3 + a + 1$, *Ans.*

29. $x^2 + 2xy + y^2$
$x + y.$
$x^3 + 2x^2y + xy^2$
$x^2y + 2xy^2 + y^3$
$x^3 + 3x^2y + 3xy^2 + y^3$, *Ans.*

30. $a^m + b^n$
$a^m + b^n$
$a^{2m} + a^m b^n$
$a^m b^n + b^{2n}$
$a^{2m} + 2a^m b^n + b^{2n}$, *Ans.*

31. $x + y + z$
$x + y + z$
$x^2 + xy + xz$
$xy + y^2 + yz$
$xz + yz + z^2$
$x^2 + 2xy + 2xz + y^2 + 2yz + z^2$, *Ans.*

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NOTES.—1. For the details in the following examples the teacher is referred to the formulas in the preceding articles in the text. (Arts. 101–103.)

2. The learner should be able to write the following answers by means of the preceding formulas.

1. $(a+1)(a+1) = a^2 + 2a + 1$, *Ans.*
2. $(2a+1)(2a+1) = 4a^2 + 4a + 1$, *Ans.*
3. $(2a-b)(2a-b) = 4a^2 - 4ab + b^2$, *Ans.*
4. $(x+y)(x+y) = x^2 + 2xy + y^2$, *Ans.*
5. $(x-y)(x-y) = x^2 - 2xy + y^2$, *Ans.*
6. $(1+x)(1-x) = 1 - x^2$, *Ans.*
7. $(7y^2-y)(7y^2-y) = 49y^4 - 14y^3 + y^2$, *Ans.*
8. $(4m-3n)(4m+3n) = 16m^2 - 9n^2$, *Ans.*
9. $(x^2-y)(x^2+y) = x^4 - y^2$, *Ans.*
10. $(1-7x)(1+7x) = 1 - 49x^2$, *Ans.*
11. $(4x-1)(4x-1) = 16x^2 - 8x + 1$, *Ans.*
12. $(5b+1)(5b+1) = 25b^2 + 10b + 1$, *Ans.*
13. $(1-x)(1-x) = 1 - 2x + x^2$, *Ans.*
14. $(1+2x)(1+2x) = 1 + 4x + 4x^2$, *Ans.*
15. $(8b-3a)(8b-3a) = 64b^2 - 48ab + 9a^2$, *Ans.*
16. $(ab+cd)(ab+cd) = a^2b^2 + 2abcd + c^2d^2$, *Ans.*
17. $(3a-2y)(3a+2y) = 9a^2 - 4y^2$, *Ans.*
18. $(x^2+y)(x^2-y) = x^4 - y^2$, *Ans.*
19. $(x-y^2)(x-y^2) = x^2 - 2xy^2 + y^4$, *Ans.*
20. $(2a^2+x)(2a^2-x) = 4a^4 - x^2$, *Ans.*

PROBLEMS.

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3. Let $x =$ the number.

   Then $\frac{2x}{3} = 24$.

   Multiplying by 3, $2x = 72$.

   Dividing by 2, $x = 36$, *Ans.*

4. Let $x =$ No. of chickens.

   Then $\frac{3x}{4} = 18$.

   Multiplying by 4, $3x = 72$.

   Dividing by 3, $x = 24$ chickens, *Ans.*

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6. Let x = number.

Then $\frac{2x}{3} - \frac{x}{2} = 8.$

Multiplying by 6, $4x - 3x = 48.$

$\therefore \quad x = 48$, *Ans.*

Proof. $\frac{2 \times 48}{3} - \frac{48}{2} = 32 - 24 = 8.$

7. Let x = number of army.

Then $\frac{3x}{7} = 840.$

Multiplying by 7, $3x = 5880.$

Dividing by 3, $x = 1960$, *Ans.*

8. Let x = worth of yacht.

Then $\frac{3x}{8} = \$360.$

Multiplying by 8, $3x = \$2880.$

Dividing by 3, $x = \$960$, *Ans.*

9. Given $\frac{4x}{5} = 20.$

Multiplying by 5, $4x = 20 \times 5.$

Dividing by 4, $x = 5 \times 5 = 25$, *Ans.*

10. Given $\frac{5x}{4} = 20.$

Multiplying by 4, $5x = 20 \times 4.$

Dividing by 5, $x = 4 \times 4 = 16$, *Ans.*

11. Given $\frac{3x}{7} = 24.$

Multiplying by 7, $3x = 24 \times 7$

Dividing by 3, $x = 8 \times 7 = 56$, *Ans.*

12. Given $\frac{4x}{11} = 28.$

Multiplying by 11, $4x = 28 \times 11.$

Dividing by 4, $x = 7 \times 11 = 77$, *Ans.*

13. Let x = number of apples.

Then $\frac{5x}{6} = 30.$

Multiplying by 6, $5x = 180.$

Dividing by 5, $x = 36$ apples, *Ans.*

14. Let x = number of sheep.

Then $\frac{3x}{7} = 30$ cows.

Mult. and dividing, $x = 70$ sheep;

And $70 + 30 = 100$ animals, } *Ans.*

15. Let x = one part,

Then $\frac{3x}{4}$ = other part.

And $x + \frac{3x}{4} = 28.$

Multiplying by 4, $4x + 3x = 112.$

Uniting terms, $7x = 112.$

Dividing by 7, $x = 16,$

And $\frac{3x}{4} = 12,$ } *Ans.*

16. Let x = whole number of plums,

Then $\frac{x}{3} + \frac{x}{4}$ = number given away,

And $x - \frac{x}{3} - \frac{x}{4} = 10,$ number left.

Multiplying by 3 and 4,

$$12x - 4x - 3x = 120.$$

Uniting, dividing, etc.,

$x = 24$ plums, *Ans.*

17. Let x = number.

Then $\frac{x}{3} + \frac{x}{6} = 21.$

Mult. by 3 and 6, $6x + 3x = 378.$

Uniting, dividing, $x = 42,$ *Ans.*

x = number.

$$\frac{x}{4} - \frac{x}{6} = 12.$$

and 6, $6x - 4x = 288.$

nd dividing, $x = 144$, *Ans.*

x = one part,

$\frac{2x}{3}$ = the other.

$$x + \frac{2x}{3} = 36.$$

$$5x = 108.$$

by 5, $x = 21\frac{3}{5}$; $\frac{2x}{3} = 14\frac{2}{5}$. } *Ans.*

x = No. of bu. from one.

$\frac{3x}{7}$ = " " the other.

$$x + \frac{3x}{7} = 21.$$

$$10x = 147.$$

by 10, $x = 14\frac{7}{10}$ bushels; $\frac{3x}{7} = 6\frac{3}{10}$ " } *Ans.*

DIVISION.

7. 12. $-3c$.

13. $7b$.
14. $5d$.
15. $-6b$.
16. $7df$.
17. $-9ag$.

Case I, Page 48.

19. d^4.
20. x^6.
21. c^5.
22. z^6.
23. $4b$.
24. $\frac{2x}{y}$.
26. $8abc^2$.
27. $-6xz$.
28. $5ab$.
29. $7x^2y$.

13. Let x = number of apples.

Then $\frac{5x}{6} = 30.$

Multiplying by 6, $5x = 180.$

Dividing by 5, $x = 36$ apples, *Ans.*

14. Let x = number of sheep.

Then $\frac{3x}{7} = 30$ cows.

Mult. and dividing, $x = 70$ sheep;

And $70 + 30 = 100$ animals, } *Ans.*

15. Let x = one part,

Then $\frac{3x}{4}$ = other part.

And $x + \frac{3x}{4} = 28.$

Multiplying by 4, $4x + 3x = 112.$

Uniting terms, $7x = 112.$

Dividing by 7, $x = 16,$ }

And $\frac{3x}{4} = 12,$ } *Ans.*

16. Let x = whole number of plums,

Then $\frac{x}{3} + \frac{x}{4}$ = number given away,

And $x - \frac{x}{3} - \frac{x}{4} = 10,$ number left.

Multiplying by 3 and 4,

$12x - 4x - 3x = 120.$

Uniting, dividing, etc.,

$x = 24$ plums, *Ans.*

17. Let x = number.

Then $\frac{x}{3} + \frac{x}{6} = 21.$

Mult. by 3 and 6, $6x + 3x = 378.$

Uniting, dividing, $x = 42,$ *Ans.*

18. Let x = number.

Then $\frac{x}{4} - \frac{x}{6} = 12.$

Mult. by 4 and 6, $6x - 4x = 288.$

Uniting and dividing, $x = 144$, *Ans.*

19. Let x = one part,

Then $\frac{2x}{3}$ = the other.

And $x + \frac{2x}{3} = 36.$

Mult. by 3, $5x = 108.$

Dividing by 5, $x = 21\frac{3}{5}$;

And $\frac{2x}{3} = 14\frac{2}{5}$. } *Ans.*

20. Let x = No. of bu. from one.

Then $\frac{3x}{7}$ = " " the other.

And $x + \frac{3x}{7} = 21.$

Mult. by 7, $10x = 147.$

Dividing by 10, $x = 14\frac{7}{10}$ bushels;

And $\frac{3x}{7} = 6\frac{3}{10}$ " } *Ans.*

DIVISION.

Case I, Page 47.

3. $2ab.$
4. $5xy.$
5. $5.$
6. $2a.$
7. $5ab.$
8. $3bc.$
9. $4mn.$
12. $-3c.$
13. $7b.$
14. $5d.$
15. $-6b.$
16. $7df.$
17. $-9ag.$

Case I, Page 48.

19. $d^4.$
20. $x^6.$
21. $c^5.$
22. $z^6.$
23. $4b.$
24. $\frac{2x}{y}.$
26. $8abc^2.$
27. $-6xz.$
28. $5ab.$
29. $7x^2y.$

30. abc.
31. $2abc$.
32. $8x^2y^2z$.
33. $8a^2bc$.
34. $12d^2x^2y$.
35. $\frac{12x^2}{a}$.
36. $12x^2z^2$.
37. $11m^2n$.

Case II, Page 49.

4. $b^2 + c^2 + d^2$.
5. $3x + 5$.
6. $3bc - 1 + 4b$.
7. $2by^2 - \frac{y}{2}$.
8. $-2x + y$.
9. $y^2 + z - 1$.
10. $-5a - 4b + 6$.
11. $3ab - 3a$.
12. $-4x^2 - 3d^2 + ax$.
13. $a^2 - 5a + 2b$.
14. $1 - 5a - 9ad$.
15. $2a - 4b - 5c$.
16. $2(a + b)^2 + 3x(a + b)^2$.
17. $9x - 9y$.
18. $x(b - c) - a(b - c)$.
19. $3a^2 - 2a$.
20. $a - a^2 + a^3$.

Case III, Page 51.

3. The dividend is the square of the divisor. Hence the quotient is $x + y$, *Ans.*

4. The solution same as Ex. 3. $a - b$, *Ans.*

5. The dividend is the *cube* of the divisor. Hence, $a^2 - 2ab + b^2$, *Ans.*

6. $a + b \;)\; ac + bc + ad + bd \;(c + d$, *Ans.*
$\underline{ac + bc}$
$ad + bd$

7. $a + b \;)\; ax + bx - ad - bd \;(x - d$, *Ans.*
$\underline{ax + bx}$
$-ad - bd$

8. $x + 2y \;)\; 2x^2 + 7xy + 6y^2 \;(2x + 3y$, *Ans.*
$\underline{2x^2 + 4xy}$
$3xy + 6y^2$

9. By Art. 103, $a^2 - b^2 = (a + b)(a - b)$, and $(a + b)(a - b) \div (a + b) = a - b$, *Ans.*

10. The solution same as Ex. 9. $x + y$, *Ans.*

11. $a - b \;)\; a^3 - b^3 \;(\; a^2 + ab + b^2$, *Ans.*

$$\begin{array}{r} a^3 - a^2b \\ \hline a^2b \\ a^2b - ab^2 \\ \hline ab^2 - b^3 \\ ab^2 - b^3 \\ \hline \end{array}$$

12. $2a + 3b \;)\; 6a^2 + 13ab + 6b^2 \;(\; 3a + 2b$, *Ans.*
1st prod., $6a^2 + 9ab$, 2d div., $4ab + 6b^2$.
2d " $4ab + 6b^2$.

13. $a - 3 \;)\; a^2 - a - 6 \;(\; a + 2$, *Ans.*
1st prod., $a^2 - 3a$. 2d div., $2a - 6$.
2d " $2a - 6$.

14. $(a^3 - 3a^2x + 3ax^2 - x^3) \div (a - x) = a^2 - 2ax + x^2$, *Ans.*

NOTE.—The dividend is the *cube* of the divisor.
Hence the quotient is the square of the divisor.

15. $3x - 6 \;)\; 6x^4 - 96 \;(\; 2x^3 + 4x^2 + 8x + 16$, *Ans.*
1st prod., $6x^4 - 12x^3$. 2d div., $12x^3$.
2d " $12x^3 - 24x^2$. 3d " $24x^2$.
3d " $24x^2 - 48x$. 4th " $48x - 96$.
4th " $48x - 96$.

16. $x + 2 \;)\; x^2 + 7x + 10 \;(x + 5$, *Ans.*
1st prod., $x^2 + 2x$. 2d div., $5x + 10$.
2d " $5x + 10$.

17. $x - 3 \;)\; x^2 - 5x + 6 \;(\; x - 2$, *Ans.*
1st prod., $x^2 - 3x$. 2d div., $-2x + 6$.
2d " $-2x + 6$.

18. $(c^2 - 2cx + x^2) \div (c - x) = c - x$, *Ans.* (Art. 102.)
19. $(a^2 + 2ab + b^2) \div (a + b) = a + b$, *Ans.* (Art. 101.)
20. $22(a - b) \div 11(a - b) = 2(a - b)$, *Ans.*

PROBLEMS.

Page 51.

1. Let x = son's age.
Then $5x - 4$ = father's age.
And $6x - 4 = 56$ years.
Adding 4 to each side, $6x = 60$ "
Dividing by 6, $x = 10$ yrs., son;
And $5x - 4 = 46$ " father. } *Ans.*

2. Let x = Frank's number,
Then $3x$ = John's "
And $4x = 60$ marbles.
Dividing by 4, $x = 15$, Frank's number;
And $3x = 45$, John's " } *Ans.*

3. Let x = one,
Then $5x$ = other.
And $6x = 72$.
$\therefore\ x = 12$;
And $5x = 60$. } *Ans.*

4. Let x = No. given to one,
Then $4x - 3$ = " " " the other.
And $5x - 3 = 57$ pears.
Adding 3 to each side, $5x = 60$ "
$\therefore\ x = 12$;
And $4x - 3 = 45$. } *Ans.*

5. Let x = No. 1st had;
Then $2x$ = " 2d "
$3x + 4$ = " 3d "
And $6x + 4 = 190$ cents.
Subtracting 4 from each side, $6x = 186$ "
$\therefore\ x = 31$ cts.;
And $2x = 62$ "
" $3x + 4 = 97$ " } *Ans.*

Page 52.

6. Let x = number of cows,
Then $9x$ = " " sheep.
And $10x = 200$.
$\therefore \quad x =$ 20 cows; } *Ans.*
And $9x =$ 180 sheep. }

7. Let x = less,
Then $3x + 3$ = greater.
And $4x + 3 = 57$.
Subtracting 3, $4x = 54$.
$\therefore \quad x = 13\frac{1}{2}$, less; } *Ans.*
And $3x + 3 = 43\frac{1}{2}$, greater, }

8. Given $2x + 4x + x - 3 = 60$.
Adding 3, $7x = 63$.
$\therefore \quad x = 9$, *Ans.*

9. Let x = No. of hours each travels,
Then $4x$ = " " miles A "
$3x$ = " " " B "
And $7x = 35$ miles, the given distance.
$\therefore \quad x = 5$ hours, *Ans.*

10. Given $14a + 7 = 119$.
Subtracting 7 from each side, $14a = 112$.
$\therefore \quad a = 8$, *Ans.*

11. Given $20b - 10 = 130$.
Adding 10 to each side, $20b = 140$.
$\therefore \quad b = 7$, *Ans.*

12. Let x = No. bought of each fruit,
Then $3x$ = amount paid for pears,
And $4x$ = " " " oranges,
" $5x$ = " " " bananas.
" $12x = 60$ cents.
$\therefore \quad x = 5$, *Ans.*

13. Let x = No. of hrs. it takes both to empty it.

Then $\frac{x}{20}$ = part one will discharge in x hours,

And $\frac{x}{5}$ = " other " " "

" $\frac{x}{20} + \frac{x}{5} = 1.$

Mult. and uniting, $25x = 100.$

$\therefore \quad x = 4$ hours, *Ans.*

14. Given $x + \frac{x}{3} = 45.$

Multiplying by 3, $3x + x = 135.$

Uniting terms, $4x = 135.$

Dividing by 4, $x = 33\frac{3}{4}$, *Ans.*

15. Let x = number.

Then $\frac{x}{2} + 3 = 8.$

Subt. 3 from each side, $\frac{x}{2} = 5.$

Multiplying by 2, $x = 10$, *Ans.*

16. Let x = A's number,

Then $2x$ = B's "

And $3x$ = C's "

" $6x = 42.$

Dividing by 6, $x = 7$, A's number;
And $2x = 14$, B's "
" $3x = 21$, C's "
} *Ans*

17. If $2x$ dollars = A's money,

Then $4x$ " = B's "

And $8x$ " = C's "

$\therefore$ $14x$ " = amount of all, *Ans.*

18. Let x = 1st part,
Then $3x$ = 2d "
$4x$ = 3d "
And $8x$ = 40.
$\therefore$ x = 5, 1st part;
$3x$ = 15, 2d "
$4x$ = 20, 3d " } *Ans.*

19. Let x = A's number,
Then $2x$ = B's "
$3x$ = C's "
And $6x$ = 60.
$\therefore$ x = 10, A's number;
$2x$ = 20, B's "
$3x$ = 30, C's " } *Ans.*

20. Let x = 1st part,
Then $2x$ = 2d "
$3x$ = 3d "
And $6x$ = 48.
$\because$ x = 8, 1st part;
$2x$ = 16, 2d "
$3x$ = 24, 3d " } *Ans.*

21. Let x = the number.
Then $\frac{3x}{4} + 5 = 23$.
Subtracting 5, $\frac{3x}{4} = 18$.
Multiplying by 4, $3x = 18 \times 4$.
Dividing by 3, $x = 6 \times 4 = 24$, *Ans.*

Case III—Continued. ***Page 41.***

20. $a^3 - ab + b^2$
$a + b$
$a^3 - a^2b + ab^2$
$a^2b - ab^2 + b^3$
$a^3 + b^3$, *Ans.*

21. $a^2 - ab + b^2$
$a^2 + ab + b^2$
$a^4 - a^3b + a^2b^2$
$a^3b - a^2b^2 + ab^3$
$a^2b^2 - ab^3 + b^4$
$a^4 + a^2b^2 + b^4$, *Ans.*

22. $x^2 + x + 1$
$x^2 - x + 1$
$x^4 + x^3 + x^2$
$-x^3 - x^2 - x$
$+x^2 + x + 1$
$x^4 + x^2 + 1$, *Ans.*

23. $3x^2 - 2xy + 5$
$x^2 + 2xy - 6$
$3x^4 - 2x^3y + 5x^2$
$6x^3y - 4x^2y^2 + 10xy$
$-18x^2 + 12xy - 30$
$3x^4 + 4x^3y - 13x^2 - 4x^2y^2 + 22xy - 30$, *Ans.*

24. $4ax - 2ay$
$6ax + 3ay$
$24a^2x^2 - 12a^2xy$
$12a^2xy - 6a^2y^2$
$24a^2x^2 - 6a^2y^2$, *Ans.*

25. $d + bx$
$d + cx$
$d^2 + bdx + cdx + bcx^2$, *Ans.*

27. $x + y$
$x - y.$
$x^2 + xy$
$- xy - y^2$
$x^2 - y^2$, *Ans.*

28. $a^3 + 1$
$a + 1$
$a^4 + a$
$a^3 + 1$
$a^4 + a^3 + a + 1$, *Ans.*

29. $x^2 + 2xy + y^2$
$x + y.$
$x^3 + 2x^2y + xy^2$
$x^2y + 2xy^2 + y^3$
$x^3 + 3x^2y + 3xy^2 + y^3$, *Ans.*

30. $a^m + b^n$
$a^m + b^n$
$a^{2m} + a^m b^n$
$a^m b^n + b^{2n}$
$a^{2m} + 2a^m b^n + b^{2n}$, *Ans.*

31. $x + y + z$
$x + y + z$
$x^2 + xy + xz$
$xy + y^2 + yz$
$xz + yz + z^2$
$x^2 + 2xy + 2xz + y^2 + 2yz + z^2$, *Ans.*

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NOTES.—1. For the details in the following examples the teacher is referred to the formulas in the preceding articles in the text. (Arts. 101–103.)

2. The learner should be able to write the following answers by means of the preceding formulas.

1. $(a + 1)(a + 1) = a^2 + 2a + 1$, *Ans.*
2. $(2a + 1)(2a + 1) = 4a^2 + 4a + 1$, *Ans.*
3. $(2a - b)(2a - b) = 4a^2 - 4ab + b^2$, *Ans.*
4. $(x + y)(x + y) = x^2 + 2xy + y^2$, *Ans.*
5. $(x - y)(x - y) = x^2 - 2xy + y^2$, *Ans.*
6. $(1 + x)(1 - x) = 1 - x^2$, *Ans.*
7. $(7y^2 - y)(7y^2 - y) = 49y^4 - 14y^3 + y^2$, *Ans.*
8. $(4m - 3n)(4m + 3n) = 16m^2 - 9n^2$, *Ans.*
9. $(x^2 - y)(x^2 + y) = x^4 - y^2$, *Ans.*
10. $(1 - 7x)(1 + 7x) = 1 - 49x^2$, *Ans.*
11. $(4x - 1)(4x - 1) = 16x^2 - 8x + 1$, *Ans.*
12. $(5b + 1)(5b + 1) = 25b^2 + 10b + 1$, *Ans.*
13. $(1 - x)(1 - x) = 1 - 2x + x^2$, *Ans.*
14. $(1 + 2x)(1 + 2x) = 1 + 4x + 4x^2$, *Ans.*
15. $(8b - 3a)(8b - 3a) = 64b^2 - 48ab + 9a^2$, *Ans.*
16. $(ab + cd)(ab + cd) = a^2b^2 + 2abcd + c^2d^2$, *Ans.*
17. $(3a - 2y)(3a + 2y) = 9a^2 - 4y^2$, *Ans.*
18. $(x^2 + y)(x^2 - y) = x^4 - y^2$, *Ans.*
19. $(x - y^2)(x - y^2) = x^2 - 2xy^2 + y^4$, *Ans.*
20. $(2a^2 + x)(2a^2 - x) = 4a^4 - x^2$, *Ans.*

PROBLEMS.

Page 44.

3. Let $x =$ the number.

   Then $\frac{2x}{3} = 24$.

   Multiplying by 3, $2x = 72$.

   Dividing by 2, $x = 36$, *Ans.*

4. Let $x =$ No. of chickens.

   Then $\frac{3x}{4} = 18$.

   Multiplying by 4, $3x = 72$.

   Dividing by 3, $x = 24$ chickens, *Ans.*

Page 45.

6. Let x = number.

Then $\frac{2x}{3} - \frac{x}{2} = 8.$

Multiplying by 6, $4x - 3x = 48.$

$\therefore \quad x = 48$, *Ans.*

Proof. $\frac{2 \times 48}{3} - \frac{48}{2} = 32 - 24 = 8.$

7. Let x = number of army.

Then $\frac{3x}{7} = 840.$

Multiplying by 7, $3x = 5880.$

Dividing by 3, $x = 1960$, *Ans.*

8. Let x = worth of yacht.

Then $\frac{3x}{8} = \$360.$

Multiplying by 8, $3x = \$2880.$

Dividing by 3, $x = \$960$, *Ans.*

9. Given $\frac{4x}{5} = 20.$

Multiplying by 5, $4x = 20 \times 5.$

Dividing by 4, $x = 5 \times 5 = 25$, *Ans.*

10. Given $\frac{5x}{4} = 20.$

Multiplying by 4, $5x = 20 \times 4.$

Dividing by 5, $x = 4 \times 4 = 16$, *Ans.*

11. Given $\frac{3x}{7} = 24.$

Multiplying by 7, $3x = 24 \times 7$

Dividing by 3, $x = 8 \times 7 = 56$, *Ans.*

12. Given $\frac{4x}{11} = 28.$

Multiplying by 11, $4x = 28 \times 11.$

Dividing by 4, $x = 7 \times 11 = 77$, *Ans.*

14. 1st dividend, $a^3-a^2b+3ab^2-3b^3$ | $a^2-5ab+4b^2$ 1st divisor.

$a^3-5a^2b+4ab^2$ | $a+4b$ 1st quotient.

$4a^2b-ab^2-3b^3$

$4a^2b-20ab^2+16b^3$

Canceling $19b^2$) $19ab^2-19b^3$

2d divisor, $a-b$ | $a^2-5ab+4b^2$ 2d dividend.

2d quotient, $a-4b$ | a^2-ab

$-4ab+4b^2$

Ans. $a-b$. | $-4ab+4b^2$

15. [Solution given on next page.]

MULTIPLES.

Page 68.

3. $56a^4b^2c^3d$.
4. $80x^2y^4z^5$.
5. $90a^3b^4c^5$.
6. $420a^4b^4$.
7. $315x^4y^3z^5$.
8. $84m^3n^2y^4$.

Page 69.

12. $a^3-b^3=(a+b)(a-b)$.
$a^3-b^3=(a^2+ab+b^2)(a-b)$.
$(a^3+ab+b^2)(a+b)(a-b)=a^4+a^3b-ab^3-b^4$, *Ans.*

13. $x^3-1=(x+1)(x-1)$.
$x^2+2x+1=(x+1)(x+1)$.
$(x^2+2x+1)(x-1)=x^3+x^2-x-1$, *Ans.*

14. The *g. c. d.* is $2a-1$.
$(6a^2-a-1)\div(2a-1)=3a+1$.
$(2a^2+3a-2)\times(3a+1)=6a^3+11a^2-3a-2$, *Ans.*

15. $m^2+m-2=(m-1)(m+2)$.
$m^3-1=(m-1)(m^2+m+1)$.
$(m^3-1)(m+2)=m^4+2m^3-m-2$, *Ans.*

15. 1st dividend, $3x^5 - 10x^3 + 15x + 8$ | $x^5 - 2x^4 - 6x^3 + 4x^2 + 13x + 6$ — 1st divisor.

$3x^5 - 6x^4 - 18x^3 + 12x^2 + 39x + 18$ | 3 — 1st quotient.

$2)\ 6x^4 + 8x^3 - 12x^2 - 24x - 10$

2d divisor, $3x^4 + 4x^3 - 6x^2 - 12x - 5$ | $3x^5 - 6x^4 - 18x^3 + 12x^2 + 39x + 18$ — 2d dividend.

2d quotient, $x - 5$ | $3x^5 + 4x^4 - 6x^3 - 12x^2 - 5x$

Multiply 2d remainder by $\frac{3}{2}$ for 2d part of dividend, and continue the division,

$-10x^4 - 12x^3 + 24x^2 + 44x + 18$ — 2d remainder.

$-15x^4 - 18x^3 + 36x^2 + 66x + 27$ — 2d part of dividend.

$-15x^4 - 20x^3 + 30x^2 + 60x + 25$

$2)\ 2x^3 + 6x^2 + 6x + 2$ — 3d remainder.

3d dividend, $3x^4 + 4x^3 - 6x^2 - 12x - 5$ | $x^3 + 3x^2 + 3x + 1$ — 3d divisor.

$3x^4 + 9x^3 + 9x^2 + 3x$ | $3x - 5$ — 3d quotient.

$-5x^3 - 15x^2 - 15x - 5$

$-5x^3 - 15x^2 - 15x - 5$ | $x^3 + 3x^2 + 3x + 1$, *Ans.*

Having divided the greater by the less, we cancel the factor 2 from the 1st remainder for the 2d divisor; and multiply the 1st divisor by 3 for the 2d dividend.

Multiply the 2d remainder by $\frac{3}{2}$ and cancel the factor 2 from the 3d remainder.

REDUCTION OF FRACTIONS.

Case I, Page 74.

4. $\frac{1}{3xy}$.
5. $\frac{3ac}{d}$.
6. $\frac{1}{3}$.
7. $\frac{7abc^2}{9x^4y^3}$.
8. $\frac{a-b}{a+b}$.
9. $\frac{x+y}{x-y}$.
10. $\frac{3y-3x}{2x-2z}$.
11. $\frac{1}{x}$.
12. $\frac{x^2-y^2}{x^4-y^4} = \frac{1}{x^2+y^2}$.
13. $\frac{x}{a+x}$.
14. $\frac{1}{a-1}$.
15. $\frac{1}{x+y}$.

Case II, Page 75.

2. $a-x$.
3. $b-\frac{b^2}{a}$.
4. $b-c$.
5. $b+c+\frac{2c^2}{b-c}$.
6. $a-b$.
7. $b+\frac{2ab}{b-a}$.
8. $a+x+\frac{a}{a-x}$.
9. $3x+1-\frac{3y}{4x}$.

Case III, Page 75.

3. $\frac{4xy-b}{y}$.
4. $5d+\frac{a-c}{2b} = \frac{10bd+a-c}{2b}$.
5. $\frac{a^2+2ab+b^2+2x}{a+b}$.
6. $\frac{x^2-x}{x+1}$.
7. $\frac{12ac-a+b}{3c}$.
8. $\frac{39x^2+3a}{5x}$.

Case IV, Page 76.

2. $\frac{12mx}{6m}$.
3. $\frac{24a^2bx}{4ab}$.
4. $\frac{18ac^2+24bc^2}{6c^2}$.
5. $\frac{x^2-y^2}{x+y}$.
6. $\frac{6a^2x^2y-4bx^2y}{3a^2-2b}$.

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3. $\frac{ab}{ac}$.
4. $\frac{21a^2}{49a}$.
5. $\frac{x^2-y^2}{x^2-2xy+y^2}$.
6. $\frac{32a^3(x+y)}{8a^2(x+y)^2}$.

Case V, Page 78.

3. $\frac{2cx}{2dx}, \frac{2bd}{2dx}, \frac{d^2x}{2dx}$.

4. $\frac{ac^2y}{2c^2xy}, \frac{2bxy}{2c^2xy}, \frac{2c^2x^2}{2c^2xy}$.

5. $\frac{2a^2+2ab}{3ab+3b^2}, \frac{3bx}{3ab+3b^2}$.

6. $\frac{x^2-2xy+y^2}{x^2-y^2}$, $\frac{x^2+2xy+y^2}{x^2-y^2}$.

7. $\frac{a^2+ab}{3a}, \frac{15a-3}{3a}$.

8. $\frac{bdx}{bd}, \frac{2ad}{bd}, \frac{bc+b}{bd}$.

9. $\frac{2b^2c-2b^2d}{3b^2c-3b^2d}, \frac{3ac-3ad}{3b^2c-3b^2d}$, $\frac{3b^2c+3b^2d}{3b^2c-3b^2d}$.

10. $\frac{2bxy}{2bz}, \frac{bz}{2bz}, \frac{4az}{2bz}$.

11. $\frac{ax+ay}{2x+2y}, \frac{6x+6y}{2x+2y}$, $\frac{2x^2+2y^2}{2x+2y}$.

12. $\frac{a^2-2ax+x^2}{a^2-x^2}$, $\frac{a^2+2ax+x^2}{a^2-x^2}$.

Case VI, Page 79.

2. Least common multiple $= 4bcx$.

$$\frac{a}{2b}=\frac{2acx}{4bcx}, \quad \frac{bc}{x}=\frac{4b^2c^2}{4bcx}, \quad \frac{y}{4c}=\frac{bxy}{4bcx}.$$

3. Least common multiple $= 3abc$.

$$\frac{cd}{ab}=\frac{3c^2d}{3abc}, \quad \frac{2x}{3a}=\frac{2bcx}{3abc}, \quad \frac{xy}{ac}=\frac{3bxy}{3abc}.$$

4. Least common multiple $= 12y$.

$$\frac{a}{2}=\frac{6ay}{12y}, \quad \frac{b}{3}=\frac{4by}{12y}, \quad \frac{c}{4}=\frac{3cy}{12y}, \quad \frac{x}{y}=\frac{12x}{12y}.$$

5. Least common multiple $= 4b^2c$.

$$\frac{a^2c}{ab}=\frac{4abc^2}{4b^2c}, \quad \frac{2cd}{b^2c}=\frac{8cd}{4b^2c}, \quad \frac{x^2y}{4bc}=\frac{bx^2y}{4b^2c}.$$

6. Least common multiple $= 24a^2c$.

$$\frac{2ab}{3ac}=\frac{16a^2b}{24a^2c}, \quad \frac{3}{4}=\frac{18a^2c}{24a^2c}, \quad \frac{x}{a^2c}=\frac{24x}{24a^2c}, \quad \frac{1}{8}=\frac{3a^2c}{24a^2c}.$$

7. Least common multiple $= 2bc$.

$$\frac{2a}{4b} = \frac{ac}{2bc}, \quad \frac{cd}{bc} = \frac{2cd}{2bc}, \quad \frac{x^2y}{bcx} = \frac{2xy}{2bc}.$$

8. Least common multiple $= a^2 - b^2$.

$$\frac{a+b}{a-b} = \frac{(a+b)^2}{a^2-b^2}, \quad \frac{a-b}{a+b} = \frac{(a-b)^2}{a^2-b^2}, \quad \frac{a^2+b^2}{a^2-b^2}$$

9. Least common multiple $= 6xy(x+y)$.

$$\frac{2(x+y)}{3(x+y)} = \frac{4xy(x+y)}{6xy(x+y)}, \quad \frac{a}{xy} = \frac{6a(x+y)}{6xy(x+y)},$$

$$\frac{ab}{6(x+y)} = \frac{abxy}{6xy(x+y)}.$$

10. Least common multiple $= a^2b^2$.

$$\frac{d}{ab^2} = \frac{ad}{a^2b^2}, \quad \frac{x}{a^2b} = \frac{bx}{a^2b^2}.$$

11. Least common multiple $= ab^2c^2d$.

$$\frac{x}{ac} = \frac{b^2cdx}{ab^2c^2d}, \quad \frac{m}{b^2c} = \frac{acdm}{ab^2c^2d}, \quad \frac{y}{c^2d} = \frac{ab^2y}{ab^2c^2d}.$$

12. Least common multiple $= xy^2z$.

$$\frac{x}{y^2} = \frac{x^2z}{xy^2z}, \quad \frac{a+b}{xy} = \frac{ayz+byz}{xy^2z}, \quad \frac{d}{xz} = \frac{dy^2}{xy^2z}.$$

13. Least common multiple $= 12a^2cx^2$.

$$\frac{m+n}{3a^2} = \frac{4cmx^2+4cnx^2}{12a^2cx^2}, \quad \frac{m-n}{2ax^2} = \frac{6acm-6acn}{12a^2cx^2},$$

$$\frac{m^2}{4cx} = \frac{3a^2m^2x}{12a^2cx^2}.$$

ADDITION OF FRACTIONS.

Page 80.

3. $\frac{3ac}{2xy} + \frac{11ac}{2xy} + \frac{8ac}{2xy} + \frac{5ac}{2xy} = \frac{27ac}{2xy}$, *Ans.*

4. $\frac{7dxz}{5abc} + \frac{17dxz}{5abc} + \frac{11dxz}{5abc} + \frac{4dxz}{5abc} = \frac{39dxz}{5abc}$, *Ans.*

5. $\frac{2b+c}{x} + \frac{3b-c}{x} = \frac{5b}{x}$, *Ans.*

6. $\frac{3a+4b}{c} + \frac{4a-2b}{c} = \frac{7a+2b}{c}$, *Ans.*

Page 81.

8. Given.

9. $\frac{3a}{4} + \frac{x}{5} + \frac{y}{3} = \frac{45a+12x+20y}{60}$, *Ans.*

10. $\frac{2x}{3} + \frac{1}{2a} + \frac{3y}{4} = \frac{8ax+6+9ay}{12a}$, *Ans.*

11. $\frac{a}{b+c} + \frac{x}{b-c} = \frac{ab-ac+bx+cx}{b^2-c^2}$, *Ans.*

12. $\frac{x+y}{2xy} + \frac{x-y}{xy} = \frac{x+y+2x-2y}{2xy} = \frac{3x-y}{2xy}$, *Ans.*

13. $\frac{2+x}{y} + \frac{3+ax}{ay} = \frac{2a+2ax+3}{ay}$, *Ans.*

14. $\frac{a}{x+y} + \frac{ab}{x-y} = \frac{ax-ay+abx+aby}{x^2-y^2}$, *Ans.*

15. $\frac{cd}{3x} + \frac{2y^*}{d} + \frac{bx}{5} = \frac{5cd^2+30xy+3bdx^2}{15dx}$, *Ans.*

16. $\frac{a}{d} - \frac{2n+d}{3h} = \frac{3ah-2dn-d^2}{3dh}$, *Ans.*

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* Fractions should be reduced to the lowest terms before reducing them to a common denominator. (Art. 175, *Note.*)

17. $\frac{a}{y} + \frac{d}{-m} = \frac{-am+dy}{-my} = \frac{am-dy}{my}$, *Ans.*

18. $\frac{-x}{y} + \frac{-h}{m-n} = \frac{nx-mx-hy}{my-ny}$, *Ans.*

19. $\frac{-4}{2} + \frac{-16}{7-3} = \frac{-14+6-16}{4} = -6$, *Ans.*

20. $\frac{4a}{b} + \frac{6c}{d} - \frac{3m}{3x} = \frac{4adx+6bcx-bdm}{bdx}$, *Ans.*

Page 81—Continued.

2. $a + \frac{b}{2} + c + \frac{d}{x} = a + c + \frac{bx+2d}{2x}$, *Ans.*

3. $x + \frac{a}{b} + \frac{-x+d}{m-y} = x + \frac{am-ay-bx+bd}{bm-by}$, *Ans.*

4. $3d - \frac{xy+z}{2} + a + \frac{b-c}{1}$

$= 3d + a + b - c - \frac{xy+z}{2}$, *Ans.*

5. $5x + \frac{a}{b} + \frac{-y}{2} = 5x + \frac{2a-by}{2b}$, *Ans.*

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7. $3d + \frac{2a}{b} = \frac{3bd+2a}{b}$, *Ans.*

8. $\frac{a+b}{c} - 4y = \frac{a+b-4cy}{c}$, *Ans.*

9. $\frac{x+y}{a} - a = \frac{x+y-a^2}{a}$, *Ans.*

10. $3x + y + \frac{a-b}{x-y} = \frac{3x^2-2xy-y^2+a-b}{x-y}$, *Ans.*

11. $-a + 5b + \frac{x-y}{a-b} = \frac{x-y-a^2+6ab-5b^2}{a-b}$, *Ans.*

12. $2x + 2y + \frac{a+b}{x-1} = \frac{2x^2+2xy-2x-2y+a+b}{x-1}$, *Ans.*

SUBTRACTION OF FRACTIONS.

Page 83.

3. $\frac{9abc}{d}$.

4. $\frac{17xyz - 5cd}{a}$.

7. $\frac{a}{m} - \frac{d-b}{y} = \frac{ay - dm + bm}{my}$, *Ans.*

8. $\frac{b-d}{m} - \left(-\frac{b}{y}\right) = \frac{b-d}{m} + \frac{b}{y} = \frac{by - dy + bm}{my}$, *Ans.*

9. $\frac{a+3d}{4} - \frac{3a-2d}{3} = \frac{3a+9d-12a+8d}{12}$

$= \frac{17d-9a}{12}$, *Ans.*

Page 84.

10. $\frac{h}{m} - \left(-\frac{h+d}{y}\right) = \frac{h}{m} + \frac{h+d}{y}$

$= \frac{hy + hm + dm}{my}$, *Ans.*

11. $\frac{h}{y} - m = \frac{h-my}{y}$, *Ans.*

NOTE. $m = \frac{my}{y}$.

12. $4a + \frac{b}{c} - \left(3a - \frac{h}{d}\right) = a + \frac{b}{c} + \frac{h}{d}$

$= a + \frac{bd + ch}{cd}$, *Ans.*

13. $a + \frac{b}{2} - \frac{d-b}{3} = \frac{2a+b}{2} - \frac{d-b}{3}$

$= \frac{6a+3b-2d+2b}{6} = \frac{6a+5b-2d}{6}$, *Ans.*

14. $\dfrac{a}{b-x} - \dfrac{c}{d+y} = \dfrac{ad+ay-bc+cx}{(b-x)(d+y)}$

$= \dfrac{ad+ay-bc+cx}{bd-dx+by-xy}$, *Ans.*

15. $a - \dfrac{x}{y} - \dfrac{3d}{2} = a - \dfrac{2x}{2y} - \dfrac{3dy}{2y} = a - \dfrac{2x+3dy}{2y}$, *Ans.*

16. $\dfrac{x-y}{10} - \dfrac{a-b}{x+y} = \dfrac{x^2-y^2-10a+10b}{10x+10y}$, *Ans.*

17. $x - \dfrac{y-c}{2} - \left(\dfrac{x-y}{3} - a\right) = \dfrac{2x-y+c}{2} - \dfrac{x-y-3a}{3}$

$= \dfrac{6x-3y+3c-2x+2y+6a}{6} = \dfrac{4x-y+3c+6a}{6}$, *Ans.*

MULTIPLICATION OF FRACTIONS.

Case I, Page 85.

5. Cancel the denominator. $h + 3d$, *Ans.*

6. Cancelling, $\dfrac{ab}{\cancel{24}_{4}} \times \cancel{6} = \dfrac{ab}{4}$, *Ans.*

7. $\dfrac{2x-3y}{15c+4d} \times (3c+2d) = \dfrac{6cx-9cy+4dx-6dy}{15c+4d}$, *Ans.*

8. $a \times \dfrac{bc}{3x} \times 6x = 2abc$, *Ans.*

9. $\dfrac{a+b}{20x+25xy} \times \dfrac{5x}{1} = \dfrac{a+b}{4+5y}$, *Ans.*

10. Factor and cancel c.

$\dfrac{a+ab}{bc+c} \times 2ac = \dfrac{2a^2+2a^2b}{b+1}$, *Ans.*

11. $\dfrac{2x+3}{\cancel{5}} \times \cancel{20}^{4}x = 8x^2 + 12x$, *Ans.*

12. $\dfrac{a-b}{\cancel{6}} \times \dfrac{\cancel{12}^{2}x+\cancel{18}^{3}}{1} = (2x+3)(a-b)$

$= 2ax - 2bx + 3a - 3b$, *Ans.*

13. $\frac{abc}{d-x} \times (d-x) = abc$, *Ans.*

14. Cancelling $4x$, $\frac{a+b}{20x} \times 4x = \frac{a+b}{5}$, *Ans.*

15. Cancelling $8z - 2$.

$$\frac{2x+y}{40z-10} \times (8z-2) = \frac{2x+y}{5}, \text{ Ans.}$$

16. $\frac{3c-d}{4,\ 28} \times \frac{3}{15} = \frac{9c-3d}{4}$, *Ans.*

17. Cancelling $y - 1$, $3x(y+1) = 3xy + 3x$, *Ans.*

18. Cancelling $x + z$, $\frac{m^2}{x^2-z^2} \times \frac{x+z}{1} = \frac{m^2}{x-z}$, *Ans.*

Case II, Pages 86, 87.

3. $6xy$.

4. $\frac{dx}{ay}$.

5. $\frac{x^2-y^2}{y^2z+yz^2}$.

6. $\frac{1}{a+3x} \times \frac{3}{8} = \frac{3}{8a+24x}$, *Ans.*

7. $\frac{(a+m)h}{3x} \times \frac{4y}{(a+m)c} = \frac{4hy}{3cx}$, *Ans.*

8. $\frac{a+b}{c^2} \times \frac{cd}{x} = \frac{d(a+b)}{cx}$, *Ans.*

11. $\frac{2x-y}{4x} \times \frac{6x-2y}{y^2-2xy} = \frac{2x-y}{2,\ 4x} \times \frac{2(3x-y)}{-y(2x-y)}$

$$= \frac{3x-y}{-2xy} = \frac{y-3x}{2xy}, \text{ Ans.}$$

12. Change the signs in the multiplicand, then factor, and cancel.

$$\frac{4a-2b}{b^2-2ab}=\frac{2b-4a}{2ab-b^2}=\frac{2(b-2a)}{b(2a-b)}, \text{ and}$$

$$\frac{2(b-2a)}{b(2a-b)}\times\frac{(2a-b)}{6a}=\frac{b-2a}{3ab}, \textit{ Ans.}$$

13. Reduce the mixed quantity to a fraction; then cancel the x.

$$\frac{ax+a+b}{x}\times\frac{ax}{by}=\frac{a^2x+a^2+ab}{by}, \textit{ Ans.}$$

14. $$x+\frac{2x}{y}=\frac{xy+2x}{y}=\frac{x(y+2)}{y};$$

$$\frac{x(y+2)}{y}\times\frac{(x+y)}{x^2}=\frac{(y+2)(x+y)}{xy}$$

$$=\frac{xy+2x+y^2+2y}{xy}, \textit{ Ans.}$$

15. $$x-\frac{y^2}{x}=\frac{x^2-y^2}{x}, \text{ and } \frac{x}{y}+\frac{y}{x}=\frac{x^2+y^2}{xy};$$

$$\frac{x^2-y^2}{x}\times\frac{x^2+y^2}{xy}=\frac{x^4-y^4}{x^2y}, \textit{ Ans.}$$

16. Reducing to a fraction, $a+\frac{2a^2}{ab}=\frac{a^2b+2a^2}{ab}$;

$$\frac{a^2b+2a^2}{ab}\times\frac{2ab}{a^2}=2b+4, \textit{ Ans.}$$

Case III, Page 88.

2. $$\frac{abc}{1}\times\frac{dx}{cy}=\frac{abdx}{y}, \textit{ Ans.}$$

3. $$\frac{ad}{1}\times\frac{b+c}{xy}=\frac{abd+acd}{xy}, \textit{ Ans.}$$

4. $\frac{ax}{1} \times \frac{m+n}{4a} = \frac{mx+nx}{4}$, *Ans.*

5. $\frac{a+h}{1} \times \frac{4c}{d} = \frac{4ac+4ch}{d}$, *Ans.*

6. $\frac{3a-y}{1} \times \frac{5x}{y} = \frac{15ax-5xy}{y}$, *Ans.*

7. $\frac{x^2+1}{1} \times \frac{5b}{x-1} = \frac{5bx^2+5b}{x-1}$, *Ans.*

8. Cancel $1+a$.

$$\frac{1-a^2}{1} \times \frac{7x}{1+a} = 7x(1-a) = 7x-7ax, \text{ Ans.}$$

9. Cancel $x+y$.

$$\frac{x^2-y^2}{1} \times \frac{ac}{3(x+y)} = \frac{ac(x-y)}{3} = \frac{acx-acy}{3}, \text{ Ans.}$$

10. Cancel $a+b$. $\frac{a^2+ab}{1} \times \frac{3c}{2(a+b)} = \frac{3ac}{2}$, *Ans.*

11. $\frac{x^2+1}{1} \times \frac{2ax}{3(x-1)} = \frac{2ax^2+2ax}{3x-3}$, *Ans.*

12. Cancel $a-b$. $\frac{2xy(a-b)}{1} \times \frac{4x}{a^2-b^2} = \frac{8x^2y}{a+b}$, *Ans.*

13. Cancel $x-1$. $\frac{3a(x-1)}{1} \times \frac{2m}{x^2-1} = \frac{6am}{x+1}$, *Ans.*

14. $\frac{2ab+b^2}{1} \times \frac{xy}{4a+b} = \frac{2abxy+b^2xy}{4a+b}$, *Ans.*

15. Change the order of the terms in the denominator, and cancel $1+n$.

$$\frac{1-n^2}{1} \times \frac{1}{1+n} = 1-n, \text{ Ans.}$$

EXAMPLES.

Page 88.

1. $\frac{3c-d}{\overset{2}{\cancel{28}}_{4}} \times \frac{\overset{3}{\cancel{15}}x}{1} = \frac{9cx-3dx}{4}$, *Ans.*

2. Cancel $y-1$. $\frac{3x}{y-1} \times \frac{y^2-1}{1} = 3x(y+1)$, *Ans.*

3. Cancel x^2. $\frac{x^2y+2x^2}{xy} \times \frac{x+y}{x^2} = \frac{(y+2)(x+y)}{xy}$, or

$$= \frac{xy+2x+y^2+2y}{xy}, \text{ Ans.}$$

4. $\frac{2ax}{a} = 2x$, $\frac{3ab}{ac} = \frac{3b}{c}$, $\frac{3ac}{2ab} = \frac{3c}{2b}$;

$$\frac{2x}{1} \times \frac{3b}{c} \times \frac{3c}{2b} = 9x, \text{ Ans.}$$

5. $\frac{\overset{2}{\cancel{3}}a^2}{\underset{2}{\cancel{18}}y} \times \frac{\overset{3}{\cancel{5}}y}{\underset{3}{\cancel{9}}a} = \frac{a}{6}$, *Ans.*

6. $\frac{a^2-b^2}{a} \times \frac{a}{a+b} \times \frac{x}{a-b} = x$, *Ans.*

Page 89.

7. $\frac{m^2}{x^2-z^2} \times \frac{x+z}{1} = \frac{m^2}{x-z}$, *Ans.*

8. $\frac{3a^2}{y} \times \frac{2y^3}{1} = 6a^2y^2$, *Ans.*

9. Cancel $x-y$.

$$\frac{x+y}{x-y} \times \frac{x^2-2xy+y^2}{1} = (x+y)(x-y) = x^2-y^2.$$

10. Cancel $8z-2$.

$$\frac{2x+y}{40z-10} \times \frac{8z-2}{1} = \frac{2x+y}{5}, \text{ Ans.}$$

11. $\left(x-\frac{y^2}{x}\right)\times\left(\frac{x}{y}+\frac{y}{x}\right)=\frac{x^2-y^2}{x}\times\frac{x^2+y^2}{xy}=\frac{x^4-y^4}{x^2y}$.

12. $a+\frac{2a^2}{ab}=\frac{a^2b+2a^2}{ab}=\frac{a^2(b+2)}{ab}$;

$$\left(a+\frac{2a^2}{ab}\right)\times\frac{2ab}{a^2}=\frac{a^2(b+2)}{ab}\times\frac{2ab}{a^2}=2b+4,\ \textit{Ans.}$$

13. Cancel $c+d$ and $2a$.

$$\frac{(c+d)^2}{2a}\times\frac{4a^2}{c+d}=2a(c+d),\ \textit{Ans.}$$

14. Change the signs in the multiplier, and cancel $2x-y$ and 2. (Art. 166.)

$$\frac{2x-y}{4x}\times\frac{2y-6x}{2xy-y^2}=\frac{2x-y}{4x}\times\frac{2(y-3x)}{y(2x-y)}=\frac{y-3x}{2xy}.$$

15. $b+\frac{2bc}{b-c}=\frac{b^2-bc+2bc}{b-c}=\frac{b^2+bc}{b-c}=\frac{b(b+c)}{b-c}$;

$$b-\frac{2bc}{b+c}=\frac{b^2+bc-2bc}{b+c}=\frac{b^2-bc}{b+c}=\frac{b(b-c)}{b+c};$$

$$\left(b+\frac{2bc}{b-c}\right)\times\left(b-\frac{2bc}{b+c}\right)=\frac{b(b+c)}{b-c}\times\frac{b(b-c)}{b+c}$$

$$=b^2,\ \textit{Ans.}$$

16. $\frac{2a}{x-y}\times\frac{x^2-y^2}{ax}=\frac{2(x+y)}{x}$, *Ans.*

DIVISION OF FRACTIONS.

Case I, Page 90.

5. $\frac{6x^2y}{n}\div 3xy=\frac{2x}{n}$, *Ans.*

6. $\frac{2a^2}{3ac}\div b=\frac{2a}{3bc}$, *Ans.*

7. $\left(a+\frac{ab}{c}\right)\div a=1+\frac{b}{c}$, *Ans.*

8. $ax + \frac{axy}{a} = ax + xy$, and

$$(ax + xy) \div x^2 = \frac{a+y}{x}, \text{ Ans.}$$

9. $\frac{a(a+x)}{2b} \div (a+x) = \frac{a}{2b}$, *Ans.*

10. $\frac{a^2 - c^2}{b+c} \div (a-c) = \frac{a^2 + ac + c^2}{b+c}$, *Ans.*

11. The numerator is a square.

$$\frac{(x+y)^2}{a+c} \div (x+y) = \frac{x+y}{a+c}, \text{ Ans.}$$

12. $\frac{x+2y}{a-b} \div (a+b) = \frac{x+2y}{a^2 - b^2}$, *Ans.*

Case II, Pages 91, 92.

2. 3 times.

3. 5.

5. $\frac{ab}{cd} \times \frac{ay}{x} = \frac{a^2by}{cdx}$, *Ans.*

6. $\frac{3abx}{6aby} \times \frac{ax}{2y} = \frac{ax^2}{4y^2}$, *Ans.*

7. $\frac{3x^2}{4} \times \frac{2}{xy} = \frac{3x}{2y}$, *Ans.*

8. $\frac{xy}{x-1} \times \frac{2}{xy} = \frac{2}{x-1}$, *Ans.*

9. $\frac{a-1}{x} \times \frac{ax}{2} = \frac{a^2 - a}{2}$, *Ans.*

10. $\frac{5x^2y^2}{10ab} \times \frac{20abx}{15xy} = \frac{2x^2y}{3}$, *Ans.*

11. $\frac{12(x+y)}{ab} \times \frac{2ab}{4(x+y)} = 6$, *Ans.*

12. $\frac{3a^2b^2}{a+b} \times \frac{a}{6ab} = \frac{a^2b}{2a+2b}$, *Ans.*

13. $\frac{x+y}{2b^2c^2} \times \frac{6bc}{axy} = \frac{3x+3y}{abcxy}$, *Ans.*

14. $\frac{x-a}{4cd} \times \frac{2d}{3bc} = \frac{x-a}{6bc^2}$, *Ans.*

15. $\frac{2xy}{x+y} \times \frac{4y}{3ax} = \frac{8y^2}{3ax+3ay}$, *Ans.*

16. $\frac{2x^2y^2}{a+b} \times \frac{2y}{3xyz} = \frac{4xy^2}{3az+3bz}$, *Ans.*

17. $\frac{x^4}{ax^3} \times \frac{bx}{ax} = \frac{bx^2}{a^2}$, *Ans.*

18. $\frac{36ad}{5b} \times \frac{10by}{18ab} = \frac{4dy}{b}$, *Ans.*

19. $\frac{5b}{36ad} \times \frac{18ab}{10by} = \frac{b}{4dy}$, *Ans.*

20. $\frac{10by}{18ab} \times \frac{36ad}{5b} = \frac{4dy}{b}$, *Ans.*

Case III, Page 92—Continued.

2. $aby \div \frac{cx}{dm} = \frac{abdmy}{cx}$, *Ans.*

3. $\frac{ax}{1} \times \frac{m+n}{4a} = \frac{mx+nx}{4}$, *Ans.*

4. $\frac{a+x}{1} \times \frac{x}{5c} = \frac{ax+x^2}{5c}$, *Ans.*

5. $\frac{5x-y}{1} \times \frac{5x}{y} = \frac{25x^2-5xy}{y}$, *Ans.*

6. $\frac{x^2+1}{1} \times \frac{5a}{x+1} = \frac{5ax^2+5a}{x+1}$, *Ans.*

7. $\frac{1-a^2}{1} \times \frac{3x}{1+a} = 3x-3ax$, *Ans.*

10. $$\frac{\frac{a+1}{a-1}}{\frac{a-1}{a+1}} = \frac{a+1}{a-1} \div \frac{a-1}{a+1} = \frac{a+1}{a-1} \times \frac{a+1}{a-1} = \frac{a^2+2a+1}{a^2-2a+1}, \text{ Ans.}$$

11. $$\frac{\frac{a-b}{x+y}}{\frac{x-y}{a+b}} = \frac{a-b}{x+y} \times \frac{a+b}{x-y} = \frac{a^2-b^2}{x^2-y^2}, \text{ Ans.}$$

12. $$\frac{\frac{x^2-y^2}{a-b}}{\frac{x+y}{1}} = \frac{x^2-y^2}{1} \times \frac{x+y}{a-b} = \frac{x^2-xy^2+x^2y-y^2}{a-b}, \text{ Ans.}$$

13. $$\frac{\frac{x+y}{a-b}}{\frac{a+b}{x-y}} = \frac{x+y}{a-b} \times \frac{x-y}{a+b} = \frac{x^2-y^2}{a^2-b^2}, \text{ Ans.}$$

EXAMPLES.

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1. $\frac{15abc}{4xyz} \times \frac{1}{3bc} = \frac{5a}{4xyz}$, *Ans.*

2. $\frac{35bcd}{27y} \times \frac{1}{9xy} = \frac{35bcd}{243xy^2}$, *Ans.*

3. $\frac{16xy}{13a} \times \frac{39a}{2cd} = \frac{24xy}{cd}$, *Ans.*

4. $\frac{42ab}{x^2-y^2} \times \frac{x-y}{14b} = \frac{3a}{x+y}$, *Ans.*

5. $\frac{23xyz}{(a+b)^2} \times \frac{a+b}{17y^2} = \frac{23xz}{17y(a+b)}$, *Ans.*

6. $\frac{(x-y)^2}{3bc} \times \frac{24b^2}{(x-y)} = \frac{8b(x-y)}{c}$ *Ans.*

8. $\frac{3a}{a^2-x^2} \times \frac{a-x}{3} = \frac{a}{a+x}$, *Ans.*

9. $\frac{4c^2-8c}{x+y} \times \frac{x+y}{c^2-4} = \frac{4c(c-2)}{x+y} \times \frac{x+y}{(c+2)(c-2)}$

$= \frac{4c}{c+2}$, *Ans.*

10. $\frac{d^2-dx}{ac+ax} \times \frac{4(d+x)}{3(c-x)} = \frac{d(d-x)}{a(c+x)} \times \frac{4(d+x)}{3(c-x)}$

$= \frac{4d(d^2-x^2)}{3a(c^2-x^2)}$, *Ans.*

11. $\frac{2c^2}{a^2+c^2} \times \frac{a+c}{c} = \frac{2c}{a^2-ac+c^2}$, *Ans.*

12. $\frac{4(a^2-x^2)}{x} \times \frac{a-x}{3(a+x)} = \frac{4(a^2-2ax+x^2)}{3x}$, *Ans.*

13. $\frac{a-b}{(a+b)(a+b)} \times \frac{a+b}{a^2-b^2} = \frac{1}{a^2+2ab+b^2}$

$= \frac{1}{(a+b)^2}$, *Ans.*

14. $\frac{x}{x^2-1} \times \frac{x-1}{x+1} = \frac{x}{(x+1)(x+1)} = \frac{x}{x^2+2x+1}$, *Ans.*

15. $\frac{bc^2+bcd}{x+ax} \times \frac{1+a}{b(c+d)} = \frac{bc(c+d)}{x(1+a)} \times \frac{1+a}{b(c+d)}$

$= \frac{c}{x}$, *Ans.*

SIMPLE EQUATIONS.

Pages 97, 98.

3. Given $b - c + x = a - d$
Transposing, $x = a - b + c - d$, *Ans.*

4. Given $x + ab - c = a + b$
Transposing, $x = a + b - ab + c$, *Ans.*

6. Given $3x + a - 6 = b - 4 + 2x$.
Transposing and uniting, $x = 2 - a + b$, *Ans.*

7. Given $x - 3 + c = 2x + a - b$
Transposing and uniting,
$$-x = -b - c + a + 3$$
Changing all the signs, $x = b + c - a - 3$, *Ans.*

NOTE.—When it is more convenient, the unknown quantities may be transposed mentally to the second member and the known quantities to the first member. The correct result may then be set down in regular order, and thus save the intermediate step of changing all the signs.

8. Given $2y + bc - ad = y + 2m - 8$
Transposing, $y = ad - bc + 2m - 8$, *Ans.*

9. Given $3ab - y + d = -2y + 17$
Transposing, $y = 17 - 3ab - d$, *Ans.*

10. Given $4cd + 27 - 4x + d = 28 - 3x + 3bh$
Transposing, $4cd + 27 - 28 + d - 3bh = 4x - 3x$
Uniting, $x = 4cd + d - 3bh - 1$, *Ans.*

11. Given $b + c - 4x = 32 + b - 5x + d$
Transposing and uniting, $x = 32 - c + d$, *Ans.*

NOTE.—The b when transposed becomes minus, and cancels $+b$. Thus, $b - b = 0$.

12. Given $x + 4 - 2x - 3 = 3x + 4 + 8 - 5x$
Transposing and uniting, $x = 11$, *Ans.*

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3. Given $\frac{3x}{5} + 12 = \frac{4x}{3} + 1$

Multiplying by 5×3, $9x + 180 = 20x + 15$

Transposing and uniting, $11x = 165$

$\therefore \quad x = 15$, *Ans.*

4. Given $\frac{2x}{3} - \frac{x}{6} = 6x - 66$

Multiplying by 6, $4x - x = 36x - 66 \times 6$

Transposing and uniting, $33x = 66 \times 6$

Dividing by coefficient, $x = 2 \times 6$

$\therefore \quad x = 12$, *Ans.*

5. Given $\frac{4x}{10} + \frac{3}{5} = 35 - x$

Multiplying by 10, $4x + 6 = 350 - 10x$

Transposing and uniting, $14x = 344$

Dividing by coefficient, $x = 24\frac{4}{7}$, *Ans.*

7. Given $\frac{4a - 5b}{x} = -\frac{3b}{d}$

Multiplying by dx, $4ad - 5bd = -3bx$

Transposing, $3bx = 5bd - 4ad$

Dividing by coefficient, $x = \frac{5bd - 4ad}{3b}$, *Ans.*

8. Given $3x - \frac{7x}{5} = a - \frac{2b + c}{10}$

Mult. by 10, $30x - 14x = 10a - 2b - c$

Uniting the terms, $16x = 10a - 2b - c$

Dividing by coef., $x = \frac{10a - 2b - c}{16}$, *Ans.*

9. Given $-x + \frac{x}{3} + \frac{3x}{4} = \frac{15}{24}$

Multiplying by 24, $-24x + 8x + 18x = 15$

Uniting, $2x = 15$

Dividing, $x = 7\frac{1}{2}$, *Ans.*

[illegible] Given $\frac{x}{2} - \frac{x}{1} = 11 - \frac{x}{6}$

[illegible] $3x - 6x = 66 - x$

[illegible] $-3x = 66$

$x = 22$, Ans.

15. Given $\frac{3x-1}{2} - x = \frac{2x}{3} - \frac{x-1}{6}$

[illegible] $3x - 3 - 6x = 4x - x - 1$

Transposing, $-4x = 2$

$\therefore \quad x = 14$, Ans.

16. Given $\frac{5x}{12} - \frac{2x}{1} = \frac{4x}{15} - 1\frac{4}{15}$

Reducing to lowest terms, $\frac{5x}{12} - \frac{2x}{1} = \frac{4x}{15} - \frac{19}{15}$

Multiplying by 60, $25x - 120x = 16x - 76$

Transposing, $-95x = -76$

$\therefore \quad x = -1\frac{3}{35}$, Ans.

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15. Given $\frac{2x}{2} - 1 - \frac{2x}{3} = \frac{x}{6} + 2$

Clearing of fractions, $\frac{2x}{3} = 8$

Multiplying by 3, $2x = 24$

$\therefore \quad x = 12$, Ans.

17. Given $\frac{4x}{5} = \frac{3x}{4} + 15 - 12$

Clearing of fractions, $16x = 15x + 60$

$\therefore \quad x = 60$, Ans.

18. Given $2x - 4 = \frac{x}{2} + 2$

Uniting and mult. by 2, $4x = x + 12$

Transposing, $3x = 12$

$\therefore \quad x = 4$, Ans.

19. Given $\frac{3x}{4} + \frac{4x}{5} - 1\frac{5}{6} = \frac{3x}{5} + 17\frac{1}{6}$

Transposing, $\frac{3x}{4} + \frac{x}{5} = 19$

Clearing of fractions, $15x + 4x = 19 \times 20$

Or, $19x = 19 \times 20$

$\therefore \quad x = 20$, *Ans.*

20. Given $\frac{x}{a} = b + c$

Multiplying by a, $x = ab + ac$, *Ans.*

21. Given $\frac{ax}{n} = d$

Multiplying by n, $ax = dn$

$\therefore \quad x = \frac{dn}{a}$, *Ans.*

22. Given $\frac{ax}{2} + \frac{bx}{3} = c$

Clearing of fractions, $3ax + 2bx = 6c$

Factoring, $(3a + 2b)\,x = 6c$

$\therefore \quad x = \frac{6c}{3a + 2b}$, *Ans.*

23. Given $\frac{2ax + b}{a} = \frac{cx + d}{c}$

Clearing of fractions, $2acx + bc = acx + ad$

Transposing, $acx = ad - bc$

$\therefore \quad x = \frac{ad - bc}{ac}$, *Ans.*

24. Given $\frac{2c}{a} + \frac{b}{x} = \frac{c}{2} + \frac{c}{a}$

Transposing, $\frac{c}{a} + \frac{b}{x} = \frac{c}{2}$

Clearing of fractions, $2cx + 2ab = acx$

Transposing, $2cx - acx = -2ab$

Changing signs, etc., $(ac - 2c)\,x = 2ab$

$\therefore \quad x = \frac{2ab}{ac - 2c}$, *Ans.*

12. Given $\frac{x}{2} + \frac{2x}{6} = 16 + \frac{x}{6}$

Multiplying by 6, $3x + 2x = 96 + x$

Uniting terms, $4x = 96$

$\therefore \quad x = 24$, *Ans.*

13. Given $\frac{3x+1}{2} - 10 = \frac{2x}{3} + \frac{x-1}{6}$

Clearing of frac., $9x + 3 - 60 = 4x + x - 1$

Transposing, $4x = 56$

$\therefore \quad x = 14$, *Ans.*

14. Given $\frac{6x}{10} + \frac{5x}{6} = \frac{4x}{15} - 1\frac{4}{15}$

Reducing to lowest terms, $\frac{3x}{5} + \frac{5x}{6} = \frac{4x}{15} - \frac{19}{15}$

Mult. by 30 (l. c. m.), $18x + 25x = 8x - 38$

Uniting terms, $35x = -38$

$\therefore \quad x = -1\frac{3}{35}$, *Ans.*

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16. Given $\frac{3x}{6} - 6 + \frac{2x}{6} = \frac{x}{6} + 2$

Uniting terms, $\frac{2x}{3} = 8$

Multiplying by 3, $2x = 24$

$\therefore \quad x = 12$, *Ans.*

17. Given $\frac{4x}{5} = \frac{3x}{4} + 15 - 12$

Clearing of fractions, $16x = 15x + 60$

$\therefore \quad x = 60$, *Ans.*

18. Given $2x - 4 = \frac{x}{2} + 2$

Uniting and mult. by 2, $4x = x + 12$

Transposing, $3x = 12$

$\therefore \quad x = 4$, *Ans.*

19. Given $\frac{3x}{4} + \frac{4x}{5} - 1\frac{5}{6} = \frac{3x}{5} + 17\frac{1}{6}$

Transposing, $\frac{3x}{4} + \frac{x}{5} = 19$

Clearing of fractions, $15x + 4x = 19 \times 20$

Or, $19x = 19 \times 20$

$\therefore \quad x = 20$, *Ans.*

20. Given $\frac{x}{a} = b + c$

Multiplying by a, $x = ab + ac$, *Ans.*

21. Given $\frac{ax}{n} = d$

Multiplying by n, $ax = dn$

$\therefore \quad x = \frac{dn}{a}$, *Ans.*

22. Given $\frac{ax}{2} + \frac{bx}{3} = c$

Clearing of fractions, $3ax + 2bx = 6c$

Factoring, $(3a + 2b)\,x = 6c$

$\therefore \quad x = \frac{6c}{3a + 2b}$, *Ans.*

23. Given $\frac{2ax + b}{a} = \frac{cx + d}{c}$

Clearing of fractions, $2acx + bc = acx + ad$

Transposing, $acx = ad - bc$

$\therefore \quad x = \frac{ad - bc}{ac}$, *Ans.*

24. Given $\frac{2c}{a} + \frac{b}{x} = \frac{c}{2} + \frac{c}{a}$

Transposing, $\frac{c}{a} + \frac{b}{x} = \frac{c}{2}$

Clearing of fractions, $2cx + 2ab = acx$

Transposing, $2cx - acx = -2ab$

Changing signs, etc., $(ac - 2c)\,x = 2ab$

$\therefore \quad x = \frac{2ab}{ac - 2c}$, *Ans.*

25. Given $\frac{2a}{3}+\frac{4x}{5}+\frac{3b}{2}=\frac{5a}{6}+\frac{12b}{2}$

Clearing of frac., $20a+24x+45b=25a+180b$

Transposing, $24x=5a+135b$

$$\therefore \quad x=\frac{5a+135b}{24}, \text{ Ans.}$$

26. Given $\frac{3x}{2}+\frac{2a}{3}=\frac{4b}{5}+\frac{15c}{3}$

Clearing of frac., $45x+20a=24b+150c$

Transposing and divid., $x=\frac{24b+150c-20a}{45}$, *Ans.*

27. Given $\frac{3a+x}{x}-5=\frac{6}{x}$

Multiplying by x, $3a+x-5x=6$

Transposing, $4x=3a-6$

$$\therefore \quad x=\frac{3a-6}{4}, \text{ Ans.}$$

28. Given $\frac{x-1}{x+1}+1=\frac{1}{a}$

Clearing of frac., $ax-a+ax+a=x+1$

Uniting terms, $2ax-x=1$

Factoring, $(2a-1)x=1$

$$\therefore \quad x=\frac{1}{2a-1}, \text{ Ans.}$$

29. Given $\frac{x}{a}+\frac{x}{c-a}=\frac{a}{c}+a$

Clearing of fractions,

$$c^2x-acx+acx=a^2c-a^3+a^2c^2-a^3c$$

Factoring, $c^2x=a^2(c-a+c^2-ac)$

$$\therefore \quad x=\frac{a^2(c-a+c^2-ac)}{c^2}, \text{ Ans.}$$

30. Given $x + b = \frac{x^2}{x+b}$

Mult. by $x+b$, $x^2+2bx+b^2 = x^2$

Cancelling and transp., $2bx = -b^2$

$\therefore \quad x = -\frac{b^2}{2b} = -\frac{b}{2}$, *Ans.*

31. Given $x - a = \frac{x^2+a}{x-a}$

Multiplying by $x-a$, $x^2-2ax+a^2 = x^2+a$

Changing signs, etc., $2ax = a^2 - a$

$\therefore \quad x = \frac{a-1}{2}$, *Ans.*

32. Given $3\left(\frac{x+b}{4}\right) + \left(\frac{x-b}{3}\right) = 4\left(\frac{x-b}{3}\right)$

Transposing, etc., $\frac{3x+3b}{4} = x - b$

Multiplying by 4, $3x + 3b = 4x - 4b$

Uniting terms, $x = 7b$, *Ans.*

33. Given $\frac{5x}{9} = x - \frac{2x-56}{3}$

Multiplying by 9, $5x = 9x - 6x + 168$

Transposing, $2x = 168$

$\therefore \quad x = 84$, *Ans.*

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34. Given $\frac{3x}{4} + x - \frac{x}{2} = 25$

Clearing of fractions, $3x + 4x - 2x = 100$

Uniting terms, $5x = 100$

$\therefore \quad x = 20$, *Ans.*

35. Given $80 = 4x - \frac{x}{2} - \frac{x}{6}$

Clearing of fractions, $480 = 24x - 3x - x$

Uniting terms, $20x = 480$

$\therefore \quad x = 24$, *Ans.*

36. Given $\frac{2x+1}{3} = 2x - \frac{x+3}{4}$

Clearing of fractions, $8x + 4 = 24x - 3x - 9$

Transposing, $13x = 13$

$\therefore \quad x = 1$, *Ans.*

37. Given $10 - 2x = \frac{3x+4}{3} - \frac{24-36}{3}$

Multiplying by 3, $30 - 6x = 3x + 4 - 24 + 36$

Transposing, $9x = 14$

$\therefore \quad x = 1\frac{5}{9}$, *Ans.*

38. Given $x - 3 = 15 - \frac{x+4}{11}$

Uniting terms, $x + \frac{x+4}{11} = 18$

Multiplying by 11, $11x + x + 4 = 198$

Or, $12x = 194$

$\therefore \quad x = 16\frac{1}{6}$, *Ans.*

39. Given $x + 2 = 3x + \frac{x+8}{4} - \frac{x+6}{3}$

Clearing of frac., $12x + 24 = 36x + 3x + 24 - 4x - 24$

Transposing, $23x = 24$

$\therefore \quad x = 1\frac{1}{23}$, *Ans.*

40. Given $\frac{3x}{4} + \frac{x-4}{2} - \frac{x-10}{2} = x - 6$

Uniting terms, $\frac{3x}{4} + 3 = x - 6$

Or, $\frac{3x}{4} = x - 9$

Multiplying by 4, $3x = 4x - 36$

Transposing, $x = 36$, *Ans.*

41. Given $\frac{11x-1}{12} = \frac{5x-11}{4} - \frac{x-1}{10}$

Clearing of frac., $55x - 5 = 75x - 165 - 6x + 6$

Transposing, $14x = 154$

$\therefore \quad x = 11$, *Ans.*

42. Given $\frac{4x}{5} - \frac{7x}{10} = 120$

Multiplying by 10, $8x - 7x = 1200$

$\therefore \quad x = 1200$, *Ans.*

43. Given $x - 20 = -\frac{2x+1}{5}$

Multiplying by 5, $5x - 100 = -2x - 1$

Transposing, $7x = 99$

$\therefore \quad x = 14\frac{1}{7}$, *Ans.*

44. Given $\frac{3x-5}{2} = \frac{4-2x}{3} + 12 - x$

Clearing of frac., $9x - 15 = 8 - 4x + 72 - 6x$

Transposing, $19x = 95$

$\therefore \quad x = 5$, *Ans.*

45. Given $\frac{1-x}{6} + 10 = \frac{2x}{3} - \frac{1-3x}{2}$

Clearing of fractions, $1 - x + 60 = 4x - 3 + 9x$

Transposing, $14x = 64$

$\therefore \quad x = 4\frac{4}{7}$, *Ans.*

46. Given $\frac{x}{a+1} - \frac{x}{a-1} = b$

Clearing of frac., $ax - x - ax - x = a^2b - b$

Uniting and factoring, $-2x = b(a^2 - 1)$

Changing signs and dividing, $x = \frac{b}{2}(1 - a^2)$, *Ans.*

47. Given $\dfrac{x}{a-b} - \dfrac{2+x}{a+b} = \dfrac{c}{a^2-b^2}$

Multiplying by $a^2 - b^2$,

$$ax + bx - 2a + 2b - ax + bx = c$$

Transposing, $$2bx = 2a - 2b + c$$

$$\therefore \quad x = \frac{2a - 2b + c}{2b}.$$

NOTE. — This answer may be expressed in various forms, as:

$$\begin{cases} x = \dfrac{a-b}{b} + \dfrac{c}{2b}, \text{ or} \\ x = \dfrac{2a+c}{2b} - 1, \text{ or} \\ x = \dfrac{a}{b} + \dfrac{c}{2b} - 1, \textit{ Ans.} \end{cases}$$

48. Given $\dfrac{3a+x}{x} = 5 + \dfrac{6}{x}$

Multiplying by x, $3a + x = 5x + 6$

Transposing, $4x = 3a - 6$

$$\therefore \quad x = \frac{3a-6}{4}, \textit{ Ans.}$$

49. Given $\dfrac{bx}{2} = d - \dfrac{bx}{3}$

Clearing of fractions, $3bx = 6d - 2bx$

Transposing, $5bx = 6d$

$$\therefore \quad x = \frac{6d}{5b}, \textit{ Ans.}$$

50. Given $8a = \dfrac{1-x}{1+x}$

Multiplying by $1 + x$, $8a + 8ax = 1 - x$

Transposing, $8ax + x = 1 - 8a$

Factoring, $(1 + 8a)\, x = 1 - 8a$

$$\therefore \quad x = \frac{1-8a}{1+8a}, \textit{ Ans.}$$

51. Given $\frac{x^2+4x+4}{x+2} = \frac{4ab}{16b}$

Reducing to lowest terms, $x + 2 = \frac{a}{4}$

Transposing, $x = \frac{a}{4} - 2$, *Ans.*

PROBLEMS.

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2. Let x = value of vest,
Then $4x$ = " " coat.
And $5x$ = \$40.
$\therefore$ x = \$8, vest; $4x$ = \$32, coat, } *Ans.*

3. Let x = amount paid A,
Then $2x$ = " " B,
And $3x$ = " " C.
Adding, $6x$ = \$9000.
$\therefore$ x = \$1500, A received; $2x$ = \$3000, B " $3x$ = \$4500, C " } *Ans.*

4. Let x = number of men,
Then $2x$ = " " boys,
$22x$ = " " women.
Adding, $25x$ = 1000.
$\therefore$ x = 40 men; $2x$ = 80 boys; $22x$ = 880 women, } *Ans.*

5. Let x = distance one runs,
Then $2x$ = " the other runs.
And $3x$ = 120 miles.
$\therefore$ x = 40 miles; $2x$ = 80 " } *Ans.*

6. Let $2x$ = number of barrels.
Then $10x$ = cost of one kind.
$8x$ = " " the other kind.
Adding, $18x$ = \$1200.
Dividing by 9, $2x$ = $133\frac{1}{3}$ barrels, *Ans.*

7. Let x = number 1st receives,
Then $2x$ = " 2d "
$5x$ = " 3d "
Adding, $8x$ = 96 pears.
$\therefore$ x = 12 pears, 1st;
$2x$ = 24 " 2d;
$5x$ = 60 " 3d, } *Ans.*

8. Let $12x$ = length of post.
Then $3x+4x+12 = 12x$.
Uniting terms, $5x = 12$.
$\therefore$ $x = \frac{12}{5}$.
$12x = \frac{144}{5} = 28\frac{4}{5}$ feet, *Ans.*

9. Let $20x$ = sum at first.
Then $20x-5x-3x$ = \$72.
Uniting terms, $12x$ = \$72.
$\therefore$ x = \$6.
$20x$ = \$120, *Ans.*

10. Let x = B's share,
Then $2x$ = A's "
$3x$ = C's "
Adding, $6x$ = \$300.
$\therefore$ x = \$50, B's share;
$2x$ = \$100, A's " } *Ans.*
$3x$ = \$150, C's "

11. Let x = age of wife,
Then $2x$ = " " man.
And $2x + 18 : x + 18 :: 3 : 2$.
Changing to an equation, $4x + 36 = 3x + 54$
Transposing, x = 18 years, wife's age;
$2x$ = 36 " man's " } *Ans.*

12. Let x = amount each invests.
Then $x + \$1260 = (x - \$870)\,2$.
Or, $x + \$1260 = 2x - \$1740$.
Transposing, $x = \$3000$, *Ans.*

13. Let x = one number,
Then $x + 25$ = other "
And $2(2x + 25) = 114$.
Dividing by 2, $2x + 25 = 57$.
Transposing, $2x = 32$.
$\therefore\ x = 16$, one number;
$x + 25 = 41$, other " } *Ans.*

14. Let $60x$ = amount he had at first.
Then $60x - 20x - 15x - 12x - 10x = \$300$.
Uniting terms, $3x = \$300$.
$\therefore\ x = \$100$.
$60x = \$6000$, *Ans.*

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15. Let x = the number.
Then $3x - 17 = 22$.
Transposing, $3x = 39$.
$\therefore\ x = 13$, *Ans.*

16. Let x = number of days they worked.
Then $9x + 6x = 450$.
Or, $15x = 450$.
$\therefore\ x = 30$ days, *Ans.*

17. Let x = No. of hours each is on the road.
Then $40x$ = No. of miles one travels,
$30x$ = " " " the other travels.
Adding, $70x = 420$ miles.
$\therefore\ x = 6$ hours.
$40x = 240$ miles;
$30x = 180$ " } *Ans.*

18. Let $3x$ = one part,
Then $4x$ = other part.
And $7x$ = 28 inches.
$\therefore$ x = 4 "
$3x$ = 12 inches; } *Ans.*
$4x$ = 16 " }

19. Let x = Henry's money;
Then $7x$ = Charles' money.
And $8x$ = \$200.
$\therefore$ x = \$25, Henry; } *Ans.*
$7x$ = \$175, Charles, }

20. Let x = the time past midnight.
Then $x + \frac{3}{7}x = 12$.
Clearing of fractions, $7x + 3x = 84$.
Or, $10x = 84$.
$\therefore$ $x = 8\frac{4}{10}$ hours,
or 8 hr. 24 min. A. M., *Ans.*

21. Let x = number of days.
Then $\frac{1}{20}$ = part A does in 1 day,
$\frac{x}{20}$ = " " " x days.
By conditions, $\frac{x}{20} + \frac{x}{30} + \frac{x}{40} = 1$.
Clearing of frac., $6x + 4x + 3x = 120$.
Uniting terms, $13x = 120$.
$\therefore$ $x = 9\frac{3}{13}$ days, *Ans.*

22. Let x = number of each.
Then $100x$ = amount received for horses,
$45x$ = " " " cows,
$5x$ = " " " sheep.
Adding, $150x$ = \$4800.
x = 32, the number of each, *Ans.*

23. Let $2x$ = number 1st receives.
Then $5x$ = " 2d "
$3x$ = " 3d "
Adding, $10x$ = 150 oranges.
$\therefore$ x = 15 "
$2x$ = 30 oranges, 1st;
$5x$ = 75 " 2d; } *Ans.*
$3x$ = 45 " 3d,

24. Let x = price of 1 chicken.
Then $3x$ = " " 1 goose.
$6x$ = " " 1 turkey.
By conditions, $12x$ = " " 4 geese.
And $18x$ = " " 3 turkeys.
" $10x$ = " " 10 chickens.
Adding, $40x$ = \$10.00.
$\therefore$ x = \$0.25, pr. of a chicken;
$3x$ = \$0.75, " " goose; } *Ans.*
$6x$ = \$1.50, " " turkey,

25. Let x = length of fish, in inches.
4 in. = " " head,
48 in. = " " tail,
Then $\frac{x}{2}$ in. = " " body.
Adding, $\frac{x}{2} + 52$ in. = x inches.
Mult. by 2, $x + 104$ in. = $2x$ "
$\therefore$ x = 104 in. = 8 ft. 8 in., *Ans.*

26. Let x = one part,
Then $20 + x$ = other part.
Adding, $20 + 2x$ = 100.
$2x$ = 80.
$\therefore$ x = 40, one part; } *Ans.*
$20 + x$ = 60, other " }

27. Let $x =$ less,

Then $a - x =$ greater.

And $\frac{a-x}{c} = \frac{x}{d}$.

Clearing of fractions, $ad - dx = cx$.

Transposing, $cx + dx = ad$.

Factoring, $(c + d)\,x = ad$.

Dividing, $x = \frac{ad}{c+d}$.

$$a - x = a - \frac{ad}{c+d} = \frac{ac + ad - ad}{c+d};$$

Reducing, $\frac{ac + ad - ad}{c+d} = \frac{ac}{c+d}$.

$$\text{Less} = \frac{ad}{c+d}; \quad \text{Greater} = \frac{ac}{c+d}, \quad \textit{Ans.}$$

28. Let $12x =$ A's money.

Then $3x = \frac{1}{4}$ " "

" $8x = \frac{2}{3}$ " "

" $2x = \frac{1}{6}$ " "

Adding, $13x = \$1222$.

$\therefore$ $x = \$94$.

$12x = \$1128$, *Ans.*

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30. Let $x =$ number lbs. of beef,

Then $2x =$ " " mutton.

Hence, $25x =$ cost of beef.

" $40x =$ " " mutton.

Adding, $65x = \$39$, cost of both.

$\therefore$ $x =$ 60 lbs. beef;
$2x =$ 120 " mutton, } *Ans.*

31. Let x = B's age,
Then $2x$ = A's "
Hence, $3x = 2x + 15$.
$\therefore\ x = 15$ yrs., B's age;
$2x = 30$ " A's " } *Ans.*

32. Let x = C's age,
Then $x + 5$ = B's "
And $x + 8$ = A's "
The sum of the ages, $3x + 13 = 110$ yrs.
Transposing, $3x = 97$ "
$\therefore\ x = 32\frac{1}{3}$ yrs., C;
$x + 5 = 37\frac{1}{3}$ " B;
$x + 8 = 40\frac{1}{3}$ " A, } *Ans.*

33. Let x = votes for defeated candidate,
Then $x + 150$ = " " successful "
Hence, $150 + 2x = 2500$.
Transposing, $2x = 2350$.
$\therefore\ x = 1175$ votes, defeated c.;
And $x + 150 = 1325$ " successful c., } *Ans.*

34. Let x = number of artillery,
Then $3x - 20$ = " " cavalry,
$3x - 20 + 92$ = " " infantry.
Hence, $7x - 40 + 92 = 1200$.
Transposing, $7x = 1148$.
$\therefore\ x = 164$, artillery;
$3x - 20 = 472$, cavalry;
$3x - 20 + 92 = 564$, infantry, } *Ans.*

35. Let x = B's share,
Then $x + \$100$ = A's "
$x + \$300$ = C's "
Adding, $3x + \$400 = \$2000$.
Transposing, $3x = \$1600$.
$\therefore\ x = \$533\frac{1}{3}$, B's share;
$x + \$100 = \$633\frac{1}{3}$, A's "
$x + \$300 = \$833\frac{1}{3}$, C's " } *Ans.*

36. Let $3x$ = share of one,
Then $5x$ = " " the other.
Hence, $8x$ = \$150.00.
$\therefore$ x = \$18.75.
$3x$ = \$56.25, one;
$5x$ = \$93.75, other, } *Ans.*

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37. Let x = price of one horse,
Then $\$616 - x$ = " " the other horse.
Hence, $5x = 6 \times \$616 - 6x$.
Transposing, $11x = 6 \times \$616$.
$\therefore$ $x = 6 \times \$56 = \$336$, price of one;
$\$616 - x = \$280$, " " other, } *Ans.*

38. Let x = age of youngest,
Then $x + 2$ = " " next older,
And $x + 4$ = " " eldest.
Adding, $3x + 6 = 48$.
$\therefore$ x = 14 yrs., youngest;
$x + 2$ = 16 " next;
$x + 4$ = 18 " eldest, } *Ans.*

39. Suppose A and B are the messengers.
Let x = days B travels,
And $x + 5$ = " A "
Then $65x$ = distance B "
And $50x + 250$ = " A "
Hence, $50x + 250 = 65x$.
$15x = 250$.
$\therefore$ $x = 16\frac{2}{3}$ days, *Ans.*

PROOF. $50 \times 16\frac{2}{3} + 250 = 1083\frac{1}{3}$ miles.
$65 \times 16\frac{2}{3} = 1083\frac{1}{3}$ "

40. Let x = number.
Then $(x + 75)\frac{2}{3} = 250.$
Dividing by $\frac{2}{3}$, $x + 75 = 625.$
Transposing, $x = 550$, *Ans.*

41. Let $5x$ = one part,
Then $3x$ = other part.
Adding, $8x = 48.$
$\therefore \quad x = 6.$
$5x = 30;$ $3x = 18,$ } *Ans.*

42. Let $12x$ = quantity.
Then $6x + 4x + 3x = 13x = a.$
Therefore, $x = \frac{a}{13}.$
And $12x = \frac{12a}{13}$, *Ans.*

43. Let $5x$ = A's acres,
Then $7x$ = B's "
Adding, $12x = 540.$
$\therefore \quad x = 45.$
$5x = 225$ acres, A's share; $7x = 315$ " B's " } *Ans.*

44. Let x = number of hours.

Consider the cistern a unit, or 1. Since in 1 hr. 1 faucet will empty $\frac{1}{6}$ of it, another $\frac{1}{10}$, another $\frac{1}{12}$, in x hours, all will empty $\frac{x}{6} + \frac{x}{10} + \frac{x}{12}$. Therefore,

$$\frac{x}{6} + \frac{x}{10} + \frac{x}{12} = 1.$$

Clearing of fractions, $10x + 6x + 5x = 60.$
Uniting, $21x = 60.$
$\therefore \quad x = 2\frac{6}{7}$ hours, *Ans.*

45. Let $x - 1 =$ 1st part,
Then $x + 2 =$ 2d "
And $\frac{x}{3} =$ 3d "
" $4x =$ 4th "

Adding, $6x + \frac{x}{3} + 1 = 39.$
Transposing, etc., $19x = 3 \times 38.$
Or, dividing, $x = 3 \times 2.$
$\therefore \quad x = 6.$
Then $x - 1 = 5$, 1st part;
$x + 2 = 8$, 2d "
$\frac{x}{3} = 2$, 3d "
$4x = 24$, 4th "
} *Ans.*

46. Let $x =$ number.
Then $6x + 12 = 66$, the sum.
Transposing, $6x = 54.$
$\therefore \quad x = 9$, *Ans.*

47. Let $x =$ whole No. of sheep.
Then $x - 7 =$ remainder.
$\frac{\$94}{x} =$ cost of one.
And $\frac{\$94}{x} \times \frac{x-7}{4} = \$20.$
Clearing of frac., $94x - 658 = 80x.$
Transposing, $14x = 658.$
$\therefore \quad x =$ 47 sheep, *Ans.*

48. Let $4x =$ income.
Then $3x =$ A's expenses 1 yr.
And $3x + \$50 =$ B's expenses 1 year.
By conditions, $20x + \$100 = 15x + \$250.$
Transposing, $5x = \$150.$
$\therefore \quad x = \$30. \qquad 4x = \$120$, *Ans.*

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49. Let x = number of minutes required.

Considering the volume of the cistern unity, or 1, it follows: If the supply pipe will fill it in 20 min., in 1 min. it will fill $\frac{1}{20}$, and in x min., $\frac{x}{20}$ of it. In like manner, the discharge pipe will empty $\frac{1}{15}$ of it in 1 minute, and $\frac{x}{15}$ in x minutes.

Then $\frac{x}{15} - \frac{x}{20} = 1.$

Multiplying by 60, $4x - 3x = 60.$

$\therefore \quad x = 60$ min., *Ans.*

Or, let x = number of hours required to empty it.

Since one empties $\frac{1}{15}$ of the cistern while the other fills $\frac{1}{20}$ of it in 1 minute, the former gains $\frac{1}{60}$ per minute in the discharge. If to gain $\frac{1}{60}$ requires 1 minute, to gain $\frac{60}{60}$ must require 60 times 1 minute, or 60 minutes.

For, $\frac{1}{60} : \frac{60}{60} :: 1$ min. $: x$ min.

$\therefore \quad x = 60$ minutes, *Ans.*

50 Let x = number.

Then $mx - nx = d.$

Factoring and dividing, $x = \frac{d}{m-n}$, *Ans.*

51. Let $3x$ = No. leaps of Greyhound.

Then $4x$ = " " Hare takes after G. starts.

$50 + 4x$ = whole distance H. goes.

Again, 2 leaps of G. = 3 leaps of H.

Hence, 1 leap " = $\frac{1}{2}$ of 3 " "

Or, 1 " " = $\frac{3}{2}$ " "

And $3x$ leaps " = $\frac{9x}{2}$ " "

Now we have two expressions, in both of which the hare's leap is the unit of measure.

Therefore, $50 + 4x = \frac{9x}{2}$.

Multiplying by 2, $100 + 8x = 9x$.

$\therefore \quad x = 100$.

And $3x = 300$ leaps, *Ans.*

Or thus, Let x = No. of Greyhound's leaps.

Then $\frac{4x}{3}$ = " Hare's l. after G. starts.

But 2 leaps of G. = 3 leaps of H.

Hence, $x = \frac{3x}{2}$ " "

By the problem, Hare's number exceeds G.'s by 50 leaps.

Therefore, $\frac{3x}{2} - \frac{4x}{3} = 50$.

Reducing, $x = 300$ leaps, *Ans.*

52. Let ax = one,

And cx = other.

Then $ax - cx = b$.

Factoring and dividing, $x = \frac{b}{a - c}$.

And $ax = \frac{ab}{a - c}$, one;

$cx = \frac{bc}{a - c}$, other, } *Ans.*

53. Let $2x$ = weight of body in lbs.

Then $x + 9$ = " " head.

By conditions, $x + 18 = 2x$.

$\therefore \quad x = 18$.

Hence, $2x = 36$ lbs., body.

$x + 9 = 27$ " head.

9 " tail.

Adding, 72 lbs., *Ans.*

54. Let x = number of hours 2d travels,

Then $12 + x$ = " " " 1st "

$\frac{13}{2}$ = miles 1st travels in 1 hour.

$78 + \frac{13x}{2}$ = " " " " $(12+x)$ hrs.

$\frac{26}{3}$ = " 2d " " 1 hour.

$\frac{26x}{3}$ = " " " " x hours.

And $78 + \frac{13x}{2} = \frac{26x}{3}$.

$6 \times 78 + 39x = 52x$.

Transposing, $13x = 6 \times 78$.

$\therefore \quad x = 6 \times 6 = 36$ hours;

And $\frac{26x}{3} = 312$ miles, } *Ans.*

Or, more concisely, Let x = No. hrs. 2d travels,

Then $x + 12$ = " 1st "

Hence, $(x+12)\frac{13}{2} = \frac{13x+156}{2}$ = dist. 1st travels.

And $\frac{26x}{3}$ = " 2d "

By the conditions, $\frac{26x}{3} = \frac{13x + 156}{2}$.

Clearing of fractions, $52x = 39x + 468$.

$\therefore \quad x = 36$ hours;

$\frac{26x}{3} = 312$ miles, } *Ans.*

55. Let x = son's age,

Then $2x$ = father's age.

By conditions, $2x - 10 = 3x - 30$.

$\therefore \quad x = 20$ yrs., son;

And $2x = 40$ " father, } *Ans.*

56. Let $28x$ = number.
Then $7x - 4x = 30$.
$3x = 30$.
$\therefore \quad x = 10$.
$28x = 280$, *Ans.*

57. Let $9x$ = share of 1st,
Then $3x$ = " " 2d,
And $4x$ = " " 3d.
Adding, $16x = \$576$.
$\therefore \quad x = \$36$.
$9x = \$324$, 1st;
$3x = \$108$, 2d;
$4x = \$144$, 3d, } *Ans.*

58. See Book.

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59. Let $2x$ = 1st part,
Then $3x$ = 2d "
And $4x$ = 3d "
Adding, $9x = 36$, the number.
$\therefore \quad x = 4$.
$2x = 8$, 1st part;
$3x = 12$, 2d "
$4x = 16$, 3d " } *Ans.*

60. Let $3x$ and $4x$ = the parts.
Then $3x + 4x = 21$ inches.
$\therefore \quad x = 3$ "
$3x = 9$ inches, one part;
$4x = 12$ " other, } *Ans.*

61. Let $4x + \$15$ = her stock.
Subtracting rent, $\$5$
Taking $\frac{1}{2}$, $2)\ 4x + \$10$
$2x + \$5$ = 1st month's gain.
Adding $6x + \$15$ = stock end 1st mo.
Subtracting rent, $\$5$
Taking $\frac{1}{2}$, $2)\ 6x + \$10$
$3x + \$5$ = 2d month's gain.
By the conditions, $9x + \$15 = 8x + \$30$.
$\therefore \quad x = \$15$.
$4x + \$15 = \$75$, *Ans.*

62. Let x = No. of days he worked,
Then $60 - x$ = " " was absent.
$75x$ = earnings.
$(60 - x)\ 25$ = forfeits.
By conditions, $75x - 1500 + 25x = 1200$.
Then $100x = 2700$.
$\therefore \quad x = 27$ days, *Ans.*

63. Let $3x$ = one's share,
Then $5x$ = other's "
Adding, $8x = \$4200$.
$\therefore \quad x = \$525$.
$3x = \$1575$, share of one;
$5x = \$2625$, " " other, } *Ans.*

64. Let x = No. of days he was perfect.
Then $60 - x$ = " " failed.
By conditions, $15x - 600 + 10x = 600$.
Then $25x = 1200$.
$\therefore \quad x = 48$, days perfect.
And $60 - x = 12$ days failed, *Ans.*

65. Let $120x$ = income.
Then $40x$ = sum spent for board.
$15x$ = " " " clothing.
$12x$ = " " " charity.

By conditions, $120x - 67x = \$318$.
Uniting terms, $53x = \$318$.
$\therefore \quad x = \$6$.
$120x = \$720$, *Ans.*

66. Let $12x$ = sum.
Then $6x - \$30$ = A's share.
$4x - \$10$ = B's "
$3x + \$8$ = C's "

Adding, $13x - \$32 = 12x$.
$\therefore \quad x = \$32$.
$12x = \$384$, the sum divided;
$6x - \$30 = \$162$, A's share;
$4x - \$10 = \$118$, B's "
$3x + \$8 = \$104$, C's "
} *Ans.*

67. See Book.

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68. Let $3x$ and $4x$ = the numbers.
Then $3x + 4 : 4x + 4 :: 5 : 6$
Changing to an equation, $18x + 24 = 20x + 20$.
Transposing, $2x = 4$.
$\therefore \quad x = 2$.
$3x = 6$;
$4x = 8$,
} *Ans.*

69. Let $3x$ = greater,
Then $2x$ = less.
Adding, $5x = 5760$.
$\therefore \quad x = 1152$.
$3x = 3456$, greater; $2x = 2304$, less, *Ans.*

70. Let x = the rate of the current.
Then $9 + x$ = crew's rate down stream.
And $9 - x$ = " " up "
By conditions, $9 + x = 2(9 - x)$.
Or, $9 + x = 18 - 2x$.
Transposing, $3x = 9$.
$\therefore \quad x = 3$ miles an hour, *Ans.*

71. Let $600x$ = length of rod,
Then $60x$ = " " red,
$30x$ = " " orange,
$20x$ = " " yellow,
$15x$ = " " green,
$12x$ = " " blue,
$10x$ = " " indigo.
Hence, $600x - 147x = 302$ inches.
Uniting terms, $453x = 302$ "
$\therefore \quad x = \frac{302}{453} = \frac{2}{3}$ in.
$600x = 400$ in. $= 33\frac{1}{3}$ ft., *Ans.*

72. Let $24x$ = whole number of kings.
Then $8x$ = kings of one name.
$6x$ = " " another name.
$3x$ = " " " "
$2x$ = " " " "
Hence, $24x - 19x = 5$ kings.
$\therefore \quad x = 1$.

Hence, there were 8, 6, 3, and 2 kings of each name, respectively, *Ans.*

73. Let x = one number.
Then $x + 1$ = other "
And $x^2 + 2x + 1 - x^2 = 15$.
Uniting terms, $2x = 14$.
$\therefore \quad x = 7;$
$x + 1 = 8,$ } *Ans.*

74. (See Prob. 51.)

Let x = No. of leaps of G.

Then $80 + \frac{1}{2}x$ = " " D.

And x leaps of G. = $2x$ leaps of D.

By the conditions, $80 + \frac{1}{2}x = 2x$.

Multiplying by 2, $160 + 3x = 4x$.

$\therefore \quad x = 160.$

$\frac{1}{2}x = 240$ leaps, *Ans.*

75. The steamers will meet in any number of days which is a common multiple of 20 and 25, and the time of their first meeting in New York will be the ***l. c. m.*** of these numbers, which is 100 days.

Again, since the first makes 1 trip in 20 days, in 100 days she will make as many trips as 20 is contained times in 100, or 5 trips.

In like manner, the second will make 4 trips.

Let x = number of miles 1st sails.

Then 1 trip : 5 trips :: 6000 m. : x m.

$\therefore \quad x = 30000$ m., 1st sails;

And $\frac{4x}{5} = 24000$ " 2d " } *Ans.*

Time of meeting = 100 days, }

Or thus: Since the 1st makes a trip 5 days sooner than the 2d, it is plain she will make 5 trips while the other makes 4; now 4 times 25 days = 100 days.

Again, the second will sail $\frac{4}{5}$ as many miles as the 1st in the same time.

Let x = number of miles 1st sails.

And $\frac{4}{5}x =$ " " 2d "

Then $\frac{1}{5}x = 6000$ miles.

$\therefore \quad x = 30000$ m., 1st sails;

And $\frac{4}{5}x = 24000$ " 2d " } *Ans.*

Time of meeting = (20×5) or $(25 \times 4) = 100$ da. }

SIMULTANEOUS EQUATIONS.

TWO UNKNOWN QUANTITIES.

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2. Given $x + y = 12$ (1)
And $x - y + 4 = 8$ (2)
From (1), $x = 12 - y$ (3)
" (2), $x = 4 + y$ (4)
Equating (3) and (4), $12 - y = 4 + y$
Transposing, $2y = 8$
$\therefore \quad y = 4;$
Substituting 4 for y in (1), $x = 8,$ } *Ans.*

3. Given $3x + 2y = 48$ (1)
And $2x - 3y = 6$ (2)
From (1), $x = \frac{48 - 2y}{3}$ (3)
" (2), $x = \frac{6 + 3y}{2}$ (4)
Equating (3) and (4), $\frac{48 - 2y}{3} = \frac{6 + 3y}{2}$
Clearing of fractions, $96 - 4y = 18 + 9y$
Transposing, $13y = 78$
$\therefore \quad y = 6;$
Substituting value of y in (2), $x = 12,$ } *Ans.*

4. Given $x + y = 20$ (1)
And $2x + 3y = 42$ (2)
From (1), $x = 20 - y$ (3)
" (2), $x = \frac{42 - 3y}{2}$ (4)
Equating (3) and (4), $20 - y = \frac{42 - 3y}{2}$
Multiplying by 2, $40 - 2y = 42 - 3y$
$\therefore \quad y = 2;$
Substituting 2 in (1), $x = 18,$ } *Ans.*

5. Given $4x + 3y = 13$ (1)
And $3x + 2y = 9$ (2)
From (1), $x = \frac{13 - 3y}{4}$ (3)
" (2), $x = \frac{9 - 2y}{3}$ (4)
Equating (3) and (4), $\frac{13 - 3y}{4} = \frac{9 - 2y}{3}$
Clearing of fractions, $39 - 9y = 36 - 8y$
$\therefore \quad y = 3;$
Substituting 3 for y in (2), $x = 1,$ } *Ans.*

6. Given $3x + 2y = 118$ (1)
And $x + 5y = 191$ (2)
From (1), $x = \frac{118 - 2y}{3}$ (3)
" (2), $x = 191 - 5y$ (4)
Equating (3) and (4), $191 - 5y = \frac{118 - 2y}{3}$
Multiplying by 3, $573 - 15y = 118 - 2y$
Transposing, $13y = 455$
$\therefore \quad y = 35;$
Substituting 35 for y in (2), $x = 16,$ } *Ans.*

7. Given $4x + 5y = 22$ (1)
And $7x + 3y = 27$ (2)
From (1), $x = \frac{22 - 5y}{4}$ (3)
" (2), $x = \frac{27 - 3y}{7}$ (4)
Equating (3) and (4), $\frac{22 - 5y}{4} = \frac{27 - 3y}{7}$
Clearing of frac., $154 - 35y = 108 - 12y$
Transposing, $23y = 46$
$\therefore \quad y = 2;$
Substituting 2 for y in (3), $x = 3,$ } *Ans.*

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9. Given $x + 3y = 19$ (1)

And $5x - 2y = 10$ (2)

From (1), $x = 19 - 3y$ (3)

Substituting value of x in (2),

$5(19 - 3y) - 2y = 10$ (4)

Reducing, $95 - 15y - 2y = 10$

Transposing, etc., $17y = 85$

$\therefore \quad y = 5;$

Substituting 5 for y in (3), $x = 4,$ } *Ans.*

10. Given $\frac{x}{2} + \frac{y}{3} = 7$ (1)

And $\frac{x}{3} + \frac{y}{2} = 8$ (2)

Mult. (1) by 6, $3x + 2y = 42$ (3)

Mult. (2) by 6, $2x + 3y = 48$ (4)

From (3), $x = \frac{42 - 2y}{3}$ (5)

Substituting value of x in (4),

$\frac{84 - 4y}{3} + 3y = 48$ (6)

Mult. (6) by 3, $84 - 4y + 9y = 144$

Uniting terms, $5y = 60$

$\therefore \quad y = 12;$

Substituting 12 for y in (5), $x = 6,$ } *Ans.*

11. Given $2x + 3y = 28$ (1)

And $3x + 2y = 27$ (2)

From (1), $x = \frac{28 - 3y}{2}$ (3)

Subst. in (2), $\frac{84 - 9y}{2} + 2y = 27$

Mult. by 2, $84 - 9y + 4y = 54$

Uniting terms, $5y = 30$

$\therefore \quad y = 6;$

Substituting 6 for y in (1), $x = 5,$ } *Ans.*

12. Given $4x + y = 43$ (1)

And $5x + 2y = 56$ (2)

From (1), $y = 43 - 4x$ (3)

Subst. in (2), $5x + 86 - 8x = 56$

Uniting terms, $3x = 30$

$\therefore \quad x = 10;$

Substituting 10 for x in (3), $y = 3,$ } *Ans.*

13. Given $5x + 8 = 7y$ (1)

And $5y + 32 = 7x$ (2)

From (1), $y = \frac{5x + 8}{7}$ (3)

Subst. in (2), $\frac{25x + 40}{7} + 32 = 7x$

Mult. by 7, $25x + 40 + 224 = 49x$

Transposing, $24x = 264$

$\therefore \quad x = 11;$

Substituting 11 for x in (3), $y = 9,$ } *Ans.*

14. Given $4x + 5y = 22$ (1)

And $7x + 3y = 27$ (2)

From (1), $x = \frac{22 - 5y}{4}$ (3)

Substituting value of x in (2),

$$\frac{154 - 35y}{4} + 3y = 27$$

Clearing of fractions,

$$154 - 35y + 12y = 108$$

Transposing, $23y = 46$

$\therefore \quad y = 2;$

Substituting 2 for y in (3), $x = 3,$ } *Ans.*

Case III, Page 116.

17. Given $3x + 4y = 29$ (1)

And $7x + 11y = 76$ (2)

Multiplying (1) by 7, $21x + 28y = 203$

" (2) by 3, $21x + 33y = 228$

Subtracting, $5y = 25$

$\therefore \quad y = 5;$

Substituting 5 for y in (1), $x = 3,$ } *Ans.*

18. Given $9x - 4y = 8$ (1)

And $13x + 7y = 101$ (2)

Multiplying (1) by 7, $63x - 28y = 56$

Multiplying (2) by 4, $52x + 28y = 404$

Adding, $115x = 460$

$\therefore \quad x = 4;$

Substituting 4 for x in (1), $y = 7,$ } *Ans.*

19. Given $3x - 7y = 7$ (1)

And $12x + 5y = 94$ (2)

Multiplying (1) by 4, $12x - 28y = 28$ (3)

Subtracting, $33y = 66$

$\therefore \quad y = 2;$

Substituting 2 for y in (1), $x = 7,$ } *Ans.*

20. Given $3x + 2y = 118$ (1)

And $x + 5y = 191$ (2)

Multiplying (2) by 3, $3x + 15y = 573$ (3)

Bringing down (1), $3x + 2y = 118$

Subtracting, $13y = 455$

$\therefore \quad y = 35;$

Substituting 35 for y in (1), $x = 16.$ } *Ans.*

21. Given $4x + 5y = 22$ (1)

And $7x + 3y = 27$ (2)

Multiplying (1) by 3, $12x + 15y = 66$

Multiplying (2) by 5, $35x + 15y = 135$

Subtracting, $23x = 69$

$\therefore x = 3;$

Substituting 3 for x in (1), $y = 2,$ } *Ans.*

EXAMPLES.

1. Given $2x + 3y = 23$ (1)

And $5x - 2y = 10$ (2)

Multiplying (1) by 2, $4x + 6y = 46$

" (2) by 3, $15x - 6y = 30$

Adding, $19x = 76$

$\therefore x = 4;$

Substituting 4 for x in (2), $y = 5,$ } *Ans.*

2. Given $4x + y = 34$ (1)

And $4y + x = 16$ (2)

Multiplying (2) by 4, $4x + 16y = 64$ (3)

Subtracting (1) from (3), $15y = 30$

$\therefore y = 2;$

Substituting 2 for y in (2), $x = 8,$ } *Ans.*

3. Given $3x + 4y = 27$ (1)

And $5x + 3y = 34$ (2)

Multiplying (1) by 3, $9x + 12y = 81$ (3)

" (2) by 4, $20x + 12y = 136$

Subtracting, $11x = 55$

$\therefore x = 5;$

Substituting 5 for x in (3), $y = 3,$ } *Ans*

4. Given $2x + 7y = 34$ (1)

And $5x + 9y = 51$ (2)

Multiplying (1) by 5, $10x + 35y = 170$

Multiplying (2) by 2, $10x + 18y = 102$

Subtracting, $17y = 68$

$\therefore \quad y = 4;$

Substituting 4 for y in (1), $x = 3,$ } *Ans.*

5. Given $5x + 7y = 43$ (1)

And $11x + 9y = 69$ (2)

Multiplying (1) by 11, $55x + 77y = 473$

" (2) by 5, $55x + 45y = 345$

Subtracting, $32y = 128$

$\therefore \quad y = 4;$

Substituting 4 for y in (1), $x = 3,$ } *Ans.*

6. Given $8x - 21y = 33$ (1)

And $6x + 35y = 177$ (2)

Multiplying (1) by 3, $24x - 63y = 99$

" (2) by 4, $24x + 140y = 708$

Subtracting, $203y = 609$

$\therefore \quad y = 3;$

Substituting 3 for y in (1), $x = 12,$ } *Ans.*

7. Given $21y + 20x = 165$ (1)

And $77y - 30x = 295$ (2)

Multiplying (1) by 3, $63y + 60x = 495$

" (2) by 2, $154y - 60x = 590$

Adding, $217y = 1085$

$\therefore \quad y = 5;$

Substituting 5 for y in (1), $x = 3,$ } *Ans.*

8. Given $11x - 10y = 14$ (1)

And $5x + 7y = 41$ (2)

Multiplying (1) by 7, $77x - 70y = 98$

" (2) by 10, $50x + 70y = 410$

Adding, $127x = 508$

$\therefore x = 4;$

Substituting 4 for x in (1), $y = 3,$ } *Ans.*

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9. Given $6y - 2x = 208$ (1)

And $-4y + 10x = 156$ (2)

Multiplying (1) by 5, $30y - 10x = 1040$

Adding, $26y = 1196$

$\therefore y = 46;$

Substituting 46 for y in (2), $x = 34,$ } *Ans.*

10. Given $4x + 3y = 22$ (1)

And $5x - 7y = 6$ (2)

Multiplying (1) by 5, $20x + 15y = 110$

" (2) by 4, $20x - 28y = 24$

Subtracting, $43y = 86$

$\therefore y = 2;$

Substituting 2 for y in (2), $x = 4,$ } *Ans.*

11. Given $3x - 5y = 13$ (1)

And $2x + 7y = 81$ (2)

Multiplying (1) by 2, $6x - 10y = 26$

" (2) by 3, $6x + 21y = 243$

Subtracting, $31y = 217$

$\therefore y = 7;$

Substituting 7 for y in (1), $x = 16,$ } *Ans.*

12. Given $5x - 7y = 33$ (1)

And $11x + 12y = 100$ (2)

Multiplying (1) by 11, $55x - 77y = 363$

" (2) by 5, $55x + 60y = 500$

Subtracting, $137y = 137$

$\therefore \quad y = 1;$

Substituting 1 for y in (1), $x = 8,$ } *Ans.*

13. Given $\frac{x}{5} + \frac{y}{6} = 18$ (1)

And $\frac{x}{2} - \frac{y}{4} = 21$ (2)

Multiplying (1) by 30, $6x + 5y = 540$ (3)

" (2) by 4, $2x - y = 84$ (4)

" (4) by 3, $6x - 3y = 252$ (5)

Subtracting (5) from (3), $8y = 288$

$\therefore \quad y = 36;$

Substituting 36 for y in (4), $x = 60,$ } *Ans.*

14. Given $16x + 17y = 500$ (1)

And $17x - 3y = 110$ (2)

Multiplying (1) by 3, $48x + 51y = 1500$

" (2) by 17, $289x - 51y = 1870$

Adding, $337x = 3370$

$\therefore \quad x = 10;$

Substituting 10 for x in (2), $y = 20,$ } *Ans.*

15. Given $8x + y = 42$ (1)

And $2x + 4y = 18$ (2)

Multiplying (2) by 4, $8x + 16y = 72$ (3)

Subtracting (1) from (3), $15y = 30$

$\therefore \quad y = 2;$

Substituting 2 for y in (2), $x = 5,$ } *Ans.*

16. Given $2x + 4y = 20$ (1)

And $4x + 5y = 28$ (2)

Multiplying (1) by 2, $4x + 8y = 40$ (3)

Subtracting, $3y = 12$

$\therefore \quad y = 4;$

Substituting 4 for y in (1), $x = 2,$ } *Ans.*

17. Given $4x + 3y = 50$ (1)

And $3x - 3y = 6$ (2)

Adding, $7x = 56$

$\therefore \quad x = 8;$

Substituting 8 for x in (2), $y = 6,$ } *Ans.*

18. Given $3x + 5y = 57$ (1)

And $5x + 3y = 47$ (2)

Multiplying (1) by 5, $15x + 25y = 285$

" (2) by 3, $15x + 9y = 141$

Subtracting, $16y = 144$

$\therefore \quad y = 9;$

Substituting 9 for y in (1), $x = 4,$ } *Ans.*

19. Given $\frac{x}{2} + \frac{y}{3} = 7$ (1)

And $\frac{x}{3} + \frac{y}{4} = 5$ (2)

Multiplying (1) by 2, $x + \frac{2y}{3} = 14$ (3)

" (2) by 3, $x + \frac{3y}{4} = 15$ (4)

Subtracting (3) from (4), $\frac{3y}{4} - \frac{2y}{3} = 1$

Clearing of fractions, $9y - 8y = 12$

$\therefore \quad y = 12;$

Substituting 12 for y in (3), $x = 6,$ } *Ans.*

20. Given $2x + y = 50$ (1)

And $\frac{x}{6} + \frac{y}{7} = 5$ (2)

Multiplying (2) by 7, $\frac{7x}{6} + y = 35$ (3)

Subtracting (3) from (1), $2x - \frac{7x}{6} = 15$

Clearing of fractions, $12x - 7x = 90$

$\therefore \quad x = 18;$

Substituting 18 for x in (1), $y = 14,$ } *Ans.*

PROBLEMS.

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1. Let $x =$ one number,

$y =$ other "

Then $x + y = 70$ (1)

And $x - y = 16$ (2)

Adding $2x = 86$

$\therefore \quad x = 43;$

Substituting value of x in (1), $y = 27,$ } *Ans.*

2. Let $x =$ price of a lemon,

And $y =$ " " an orange.

Then $8x + 4y = 56$ cents. (1)

And $3x + 8y = 60$ " (2)

Mult. (1) by 2, $16x + 8y = 112$ " (3)

Subt. (2) from (3), $13x = 52$ cents.

$\therefore \quad x = 4$ cts., lem.;

Substituting 4 for x in (1), $y = 6$ " or'ge, } *Ans.*

3. Let $x =$ votes cast for one.

And $y =$ " " " the other.

Then $x + y = 375$ (1)

And $x - y = 91$ (2)

Adding, $2x = 466$

$\therefore \quad x = 233,$ v. for one;

Subst. value x in (1), $y = 142,$ " other, } *Ans.*

4. Let x = greater part,
And y = the less.
Then $x + y = 75$ (1)
And $3x - 7y = 15$ (2)
Multiplying (1) by 3, $3x + 3y = 225$ (3)
Subt. (2) from (3), $10y = 210$
$\therefore \quad y = 21$, less;
Subst. value of y in (1), $x = 54$, greater, } *Ans.*

5. Let x = price of a horse,
And y = " " cow.
Then $9x + 7y = \$1200$ (1)
And $6x + 13y = \$1200$ (2)
Mult. (1) by 2, $18x + 14y = \$2400$ (3)
" (2) by 3, $18x + 39y = \$3600$ (4)
Subt. (3) from (4), $25y = \$1200$
$\therefore \quad y = \$48$, pr. of cow;
Subst. value y in (1), $x = \$96$, " horse, } *Ans.*

6. Let x = No. of gentlemen,
And y = " ladies.
Then $y - 15$ = ladies who remained,
$x - 45$ = gent. " "
And $x = 2(y - 15)$ (1)
$5(x - 45) = y - 15$ (2)
From (1), $x = 2y - 30$ (3)
" (2), $5x - 225 = y - 15$ (4)
Substituting value of x in (4),
$5(2y - 30) - y = 210$
Transposing, $9y = 360$
$\therefore \quad y = 40$ ladies;
Subst. 40 for y in (3), $x = 50$ gentlemen, } *Ans.*

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7. Let $x = 1st$,
And $y = 2d$.
Then $5x + 2y = 19$ (1)
And $7x - 6y = 9$ (2)
Multiplying (1) by 3, $15x + 6y = 57$
Adding, $22x = 66$
$\therefore x = 3$;
Substituting 3 for x in (2), $y = 2$, } *Ans.*

8. Let $x =$ No. of men in one army,
And $y =$ " " the other.
Then $x + y = 21110$ (1)
And $2x + 3y = 52219$ (2)
Mult. (1) by 2, $2x + 2y = 42220$
Subtracting, $y = 9999$, one army;
Substituting in (1), $x = 11111$, other " } *Ans.*

9. Let $x =$ digit in tens' place,
And $y =$ " " units' "
Then $x + y = 11$ (1)
And $x + 13 = 3y$ (2)
From (1), $x = 11 - y$ (3)
Substituting value of x in (2),
$11 - y + 13 = 3y$
Transposing, $4y = 24$
$\therefore y = 6$, units' digit.
Subst. 6 for y in (3), $x = 5$, tens' "
$\therefore$ The number $= 56$, *Ans.*

10. Let $x =$ A's share, and $y =$ B's share.
Then $x + y = \$570$ (1)
And $3x + 5y = \$2350$ (2)
Mult. (1) by 3, $3x + 3y = \$1710$
Subtracting, $2y = \$640$
$\therefore y = \$320$, B's share;
Subst. 320 for y in (1), $x = \$250$, A's " } *Ans.*

11. Let $\frac{x}{y}$ = the fraction.

Then $\frac{x+1}{y} = \frac{1}{3}$, or $3x + 3 = y$ (1)

And $\frac{x}{y+1} = \frac{1}{4}$, or $4x = y + 1$ (2)

Substituting the value of y in (2), we have

$$4x = 3x + 3 + 1$$

Uniting, $x = 4$

$\therefore$ $y = 15$

And $\frac{x}{y} = \frac{4}{15}$, *Ans.*

12. Let $6x$ = A's money,

And $8y$ = B's "

Then $6x + y = \$1200$ (1)

And $x + 8y = \$2550$ (2)

Mult. (1) by 8, $48x + 8y = \$9600$

Subtracting, $47x = \$7050$

$\therefore$ $x = \$150$

$6x = \$900$, A's m.;

In (1), $y = \$300$, and $8y = \$2400$, B's " } *Ans.*

13. Let x and y denote the numbers.

Then $x - y = 14$

And $x + y = 48$

Adding $2x = 62$, $\therefore x = 31$;

Subtracting, $2y = 34$, $\therefore y = 17$, } *Ans.*

14. Let x = price of the house,

y = " " garden.

Then $x + y = \$8500$

$$y = \frac{5x}{12}$$

Substituting, $x + \frac{5x}{12} = 8500$

Mult. by 12, etc., $17x = 8500 \times 12$

$\therefore$ $x = \$6000$, house;

And $y = \$2500$, garden, } *Ans.*

15. Let $4x$ = one part,
And $6y$ = the other.
Then $4x + 6y = 50$ (1)
And $3x + 5y = 40$ (2)
Mult. (1) by 3, $12x + 18y = 150$
" (2) by 4, $12x + 20y = 160$
Subtracting, $2y = 10$ (3)
Mult. (3) by 3, $6y = 30$, one;
Substituting in (1), $4x = 20$, other, } *Ans.*

16. Let x = A's share,
And y = B's "
Then $x + y = \$1280$ (1)
And $7x = 9y$ (2)
From (2), $x = \frac{9y}{7}$
Substituting, $\frac{9y}{7} + y = \$1280$
Clearing of frac., $9y + 7y = \$1280 \times 7$
Uniting terms, $16y = \$1280 \times 7$
Dividing, $y = \$560$, B's share;
Subst. value y in (1), $x = \$720$, A's " } *Ans.*

17. Let x = age of elder,
And y = " " younger.
Then $x - y = 10$ (1)
And $x - 15 = 2(y - 15)$ (2)
Subt. (1) from (2), $y - 15 = 2y - 40$
Transposing, $y = 25$ yrs., y'nger;
Subst. 25 for y in (1), $x = 35$ " elder, } *Ans.*

18. Let x = value of 1st horse,
And y = " " 2d "
Then $x + 50 = 2y$ or $x = 2y - 50$ (1)
And $y + 50 = x - 15$ or $x = y + 65$
Equating the values of x, $2y - 50 = y + 65$
Transposing, $y = \$115$, value 2d;
Subst. value y in (1), $x = \$180$, " 1st, } *Ans.*

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19. Let x = distance steamer goes,
And y = " ship "
Then $20 + y = x$ (1)

Their respective distances for the same period of time will be in the ratio of 8 to 7. Hence, forming a proportion,

$$x : y :: 8 : 7$$

Reducing to an equation, $7x = 8y$

Dividing by 7, $x = \frac{8y}{7}$ (2)

Substituting in (1), $20 + y = \frac{8y}{7}$

Clearing of fractions, $140 + 7y = 8y$

$\therefore \quad y = 140$ miles, ship;

From (2), $x = \frac{8 \times 140}{7} = 160$ " steamer, } *Ans.*

20. Let $6x$ = greater,
And $6y$ = less.
Then $3x + 2y = 13$ (1)
And $2x - 3y = 0$ (2)
From (2), $2x = 3y$
Or, $x = \frac{3y}{2}$ (3)

Substituting in (1), $\frac{9y}{2} + 2y = 13$

Multiplying by 2, $9y + 4y = 26$

$\therefore \quad y = 2$

Substituting y in (3), $x = 3$

$6x = 18$, greater;
$6y = 12$, less, } *Ans.*

21. Let $3x$ = part left,
And $6y$ = " carried away.
Then $x + y = 28$ feet. (1)
And $15x - 36y = 12$ " (2)
Mult. (1) by 15, $15x + 15y = 420$ " (3)
Subt. (2) from (3), $51y = 408$ feet.
$\therefore \quad y = 8$ "
And $6y = 48$ " (4)
Subst. 8 for y in (1), $x = 20$ "
And $3x = 60$ " (5)
Adding (4) and (5), $3x + 6y = 108$ feet, *Ans.*

22. Let $2x$ = the lady's age,
And y = number of verses.
Then $y = x - 2$ (1)
And $2x + y = 43$ (2)
Substituting, $2x + x - 2 = 43$
Uniting terms, $3x = 45$
$\therefore \quad x = 15$
$2x = 30$ yrs., her age;
Subst. 15 for x in (1), $y = 13$ verses, } *Ans.*

23. Let x = greater,
And y = less.
Then $x - y = 20$ (1)
And $\frac{x}{y} = 3$ (2)
From (2), $x = 3y$ (3)
Substituting in (1), $3y - y = 20$
$\therefore \quad y = 10$, less;
From (3), $x = 30$, greater, } *Ans.*

24. Let x = No. of oxen, and y = No. of colts.

Then $65x + 25y = 720$ (1)

And $25x + 65y = 1440$ (2)

Mult. (1) by 13, $845x + 325y = 9360$

" (2) by 5, $125x + 325y = 7200$

$720x = 2160$

$\therefore \quad x = 3$ oxen;

Subst. in (1) and transposing, $y = 21$ colts, } *Ans.*

25. Let x = digit in tens' place,

And y = " " units' "

Then $x + y + 7 = 3x$ (1)

And $10x + y - 18 = 10y + x$ (2)

Reducing (1), $y = 2x - 7$ (3)

" (2), $x - y = 2$

Substituting, $2 - 2x + 7 = 2$

$\therefore \quad x = 5$

From (3), $y = 3$

$\therefore \quad 10x + y = 53$, *Ans.*

26. Let $6x$ = A's entire capital,

And $5y$ = B's " "

Then $6x + 5y = \$9800$ (1)

And $5x = 4y$ (2)

Or, $x = \frac{4y}{5}$

Substituting, $\frac{24y}{5} + 5y = \$9800$

Multiplying by 5, and uniting terms,

$49y = \$9800 \times 5$

$\therefore \quad y = \$1000$

And $5y = \$5000$, B's; } *Ans.*

Subst. in (1) and transp., $6x = \$4800$, A's,

27. Let x = part of purse 1 guinea fills,
And y = " " 1 dollar "
Then $6x + 19y = 1$ (1)
And $5x + 4y = \frac{17}{63}$ (2)
Multiplying (1) by 5, $30x + 95y = 5$
" (2) by 6, $30x + 24y = \frac{34}{21}$

Subtracting, $71y = 5 - \frac{34}{21} = \frac{71}{21}$
$\therefore \quad y = \frac{1}{21}$
From (2), $5x + \frac{4}{21} = \frac{17}{63}$
Transposing, $5x = \frac{17}{63} - \frac{4}{21} = \frac{5}{63}$
$\therefore \quad x = \frac{1}{63}$

Now if 1 guinea fills $\frac{1}{63}$ of the purse, it is plain that 63 guineas will fill $\frac{63}{63}$, or the entire purse. Reasoning in like manner, we find 21 silver dollars will likewise fill the purse. Hence, *Ans.* $21, or 63 guineas.

28. Let x = greater,
And y = less.
Then $x + y = a$
And $x = ny$
Substituting, $ny + y = a$
Factoring and dividing, $y = \frac{a}{n+1}$;
And $x = \frac{an}{n+1}$,
} *Ans.*

THREE OR MORE UNKNOWN QUANTITIES.

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2. Given $5x - 3y + 2z = 28$ (1)

$3x + 2y - 4z = 15$ (2)

And $-x + 3y + 4z = 24$ (3)

Adding (2) and (3), $2x + 5y = 39$ (4)

Mult. (1) by 2, $10x - 6y + 4z = 56$ (5)

Adding (2) and (5), $13x - 4y = 71$ (6)

Mult. (4) by 4, $8x + 20y = 156$ (7)

" (6) by 5, $65x - 20y = 355$ (8)

Adding (7) and (8), $73x = 511$

$\therefore x = 7;$

Substituting 7 for x in (4), $y = 5;$ } *Ans.*

" val. x and y in (2), $z = 4,$ }

3. Given $2x + 5y - 3z = 4$ (1)

$4x - 3y + 2z = 9$ (2)

And $5x + 6y - 2z = 18$ (3)

Adding (2) to (3), $9x + 3y = 27$ (4)

Mult. (1) by 2, $4x + 10y - 6z = 8$ (5)

" (2) by 3, $12x - 9y + 6z = 27$ (6)

Adding (5) to (6), $16x + y = 35$ (7)

Dividing (4) by 3, $3x + y = 9$ (8)

Subt. (8) from (7), $13x = 26$

$\therefore x = 2;$

From (8), $y = 3;$ } *Ans.*

" (2), $z = 5,$ }

4. Given $2x + 3y - 4z = 20$ (1)

$x - 2y + 3z = 6$ (2)

And $3x - 2y + 5z = 26$ (3)

Adding (1) to (2), $3x + y - z = 26$ (4)

Subt. (4) from (3), $3y - 6z = 0$

Or, $y - 2z = 0$ (5)

Bringing down (1), $2x + 3y - 4z = 20$ (1)

Mult. (2) by 2, $2x - 4y + 6z = 12$

Subtracting, $7y - 10z = 8$ (6)

Mult. (5) by 7, $7y - 14z = 0$

Subtracting, $4z = 8$

$\therefore z = 2;$

From (5), $y = 4;$ } *Ans.*

" (2), $x = 8,$ }

5. Given $5x + 2y + 4z = 46$ (1)

$3x + 2y + z = 23$ (2)

And $10x + 5y + 4z = 75$ (3)

Subt. (1) from (3), $5x + 3y = 29$ (4)

Mult. (2) by 4, $12x + 8y + 4z = 92$ (5)

Subt. (3) from (5), $2x + 3y = 17$ (6)

" (6) " (4), $3x = 12$

$\therefore x = 4;$

From (6), $y = 3;$ } *Ans.*

" (2), $z = 5,$ }

6. Given $x + y + z = 53$ (1)

$x + 2y + 3z = 105$ (2)

And $x + 3y + 4z = 134$ (3)

Subt. (1) from (2), $y + 2z = 52$ (4)

" (2) " (3), $y + z = 29$ (5)

" (5) " (4), $z = 23;$

From (5), $y = 6;$ } *Ans.*

" (1), $x = 24,$ }

7. Given $3x + 4z = 57$ (1)

$2y - z = 11$ (2)

And $5x + 3y = 65$ (3)

Multiplying (2) by 4, $8y - 4z = 44$ (4)

Adding (1) to (4), $3x + 8y = 101$ (5)

Multiplying (5) by 5, $15x + 40y = 505$ (6)

" (3) by 3, $15x + 9y = 195$

Subtracting, $31y = 310$

$y = 10;$

From (2), $z = 9;$ } *Ans.*

" (1), $x = 7,$

8. Given $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 62$ (1)

$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 47$ (4)

And $\frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 38$ (3)

Clearing of frac., $6x + 4y + 3z = 744$ (4)

$20x + 15y + 12z = 2820$ (5)

$15x + 12y + 10z = 2280$ (6)

Mult. (4) by 4, $24x + 16y + 12z = 2976$ (7)

Subt. (5) from (7), $4x + y = 156$ (8)

Mult. (5) by 5, $100x + 75y + 60z = 14100$ (9)

" (6) by 6, $90x + 72y + 60z = 13680$

Subtracting, $10x + 3y = 420$

Mult. (8) by 3, $12x + 3y = 468$

Subtracting, $2x = 48$

$\therefore x = 24;$

From (8), $y = 60;$ } *Ans.*

" (3), $z = 120,$

Page 122.

12. Given $w + x + z = 10$ (1)

$x + y + z = 12$ (2)

$w + x + y = 9$ (3)

And $w + y + z = 11$ (4)

Adding, $3w + 3x + 3y + 3z = 42$ (5)

Dividing (5) by 3, $w + x + y + z = 14$ (6)

Subtracting each given equation from (6), we have,

$w = 2$; $x = 3$; $y = 4$; and $z = 5$, *Ans.*

13. Given $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$ (1)

$\frac{1}{y} + \frac{1}{z} = \frac{7}{12}$ (2)

And $\frac{1}{x} + \frac{1}{z} = \frac{3}{4}$ (3)

Adding, $\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \frac{26}{12}$ (4)

Dividing (4) by 2, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{13}{12}$ (5)

Subtracting each given equation from (5), we have,

$\frac{1}{x} = \frac{6}{12}$; $\frac{1}{y} = \frac{4}{12}$; $\frac{1}{z} = \frac{3}{12}$;

Hence, $x = 2$; $y = 3$; $z = 4$, *Ans.*

PROBLEMS.

Page 123.

1. Let $x =$ age of 1st,

$y =$ " " 2d,

And $z =$ " " 3d.

Then $x + y = 27$ (1)

$x + z = 29$ (2)

And $y + z = 32$ (3)

Adding, $2x + 2y + 2z = 88$ (4)

Dividing (4) by 2, $x + y + z = 44$ (5)

Subtracting equations (3), (2), and (1) from (5), we have $x = 12$ yrs.; $y = 15$ yrs.; and $z = 17$ yrs., *Ans.*

2. Let $x =$ price of a calf,

$y =$ " " sheep,

And $z =$ " " lamb.

Then $7x + 13y = \$205$ (1)

$14x + 5z = \$300$ (2)

And $12y + 20z = \$140$ (3)

Mult. (1) by 2, $14x + 26y = 410$ (4)

Subt. (2) from (4), $26y - 5z = 110$ (5)

Mult. (5) by 4, $104y - 20z = 440$ (6)

Adding (3) to (6), $116y = 580$

$\therefore y = \$5$, pr. of sheep;

From (5), $z = \$4$, " lambs;

" (1), $x = \$20$, " calves, } *Ans.*

3. Let $x =$ 1st number,

$y =$ 2d "

And $z =$ 3d "

Then $x + y = 13$ (1)

$x + z = 16$ (2)

And $y + z = 19$ (3)

Adding, $2x + 2y + 2z = 48$

Dividing by 2, $x + y + z = 24$ (4)

Subt. (3) from (4), $x = 5$, 1st;

" (2) " (4), $y = 8$, 2d; } *Ans.*

" (1) " (4), $z = 11$, 3d,

4. Let $6x$ = number of men in 1st,
$3y$ = " " " 2d,
And $2z$ = " " " 3d.
Then $6x + 3y + 2z = 1905$ (1)
$3x + y = 2z - 60$ (2)
And $z + 2x = 3y - 165$ (3)
Adding (1) and (2), $9x + 4y = 1845$ (4)
Mult. (3) by 2, $4x - 6y + 2z = -330$ (5)
Subt. (5) from (1), $2x + 9y = 2235$ (6)
Mult. (6) by 9, $18x + 81y = 20115$ (7)
" (4) by 2, $18x + 8y = 3690$ (8)
Subtracting (8) from (7), $73y = 16425$
$\therefore \quad y = 225$
From (6), $x = 105$
" (2), $z = 300$
$6x = 630$, men in 1st;
$3y = 675$, " " 2d;
$2z = 600$, " " 3d, } *Ans.*

5. Let x = price of 1st,
y = " " 2d,
And z = " " 3d.
Then $12x + 13y + 14z = 25$ (1)
$10x + 17y + 11z = 24$ (2)
And $6x + 12y + 6z = 15$ (3)
Mult. (3) by 2, $12x + 24y + 12z = 30$ (4)
Subt. (1) from (4), $11y - 2z = 5$ (5)
Mult. (2) by 6, $60x + 102y + 66z = 144$ (6)
" (1) by 5, $60x + 65y + 70z = 125$ (7)
Subt. (7) from (6), $37y - 4z = 19$ (8)
Mult. (5) by 2, $22y - 4z = 10$
Subtracting, $15y = \$9.00$
$\therefore \quad y = \$0.60$, 2d;
From (5), $z = \$0.80$, 3d;
" (3), $x = \$0.50$, 1st, } *Ans.*

6. Let x = number minutes it takes A,
y = " " " B,
And z = " " " C.

Now if A can fill it in x minutes,

A can fill $\frac{1}{x}$ of it in 1 minute,

B " " $\frac{1}{y}$ " " 1 "

And C " " $\frac{1}{z}$ " " 1 "

Then $$\frac{1}{x}+\frac{1}{y}=\frac{1}{70} \quad (1)$$

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{84} \quad (2)$$

And $$\frac{1}{y}+\frac{1}{z}=\frac{1}{140} \quad (3)$$

Adding, $$\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=\frac{28}{840}=\frac{2}{60}$$

Dividing by 2, $$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{60} \quad (4)$$

Subtracting (3) from (4), $$\frac{1}{x}=\frac{140-60}{8400}$$

Or, $$\frac{1}{x}=\frac{1}{105}$$

Clearing of fractions, $x = 105$ min., A.

Subt. (2) from (4), $$\frac{1}{y}=\frac{1}{60}-\frac{1}{84}=\frac{84-60}{60\times 84}$$

Or, $$\frac{1}{y}=\frac{24}{60\times 84}=\frac{1}{210}$$

Clearing of fractions, $y = 210$ min., B.

Subtracting (1) from (4), etc., $z = 420$ min., C.

A fills it in 105 minutes;
B " " 210 "
C " " 420 "
} *Ans.*

7. Denote the parts by w, x, y, and z.

Then $w + x + y + z = 90$ (1)

$w + z = 2y$ (2)

$x - z = 2y$ (3)

And $\frac{z}{2} = 2y$ (4)

Multiplying (4) by 2, $z = 4y$ (5)

Adding (5), (3), and (2), $w + x + z = 8y$ (6)

Subtracting (6) from (1), $y = 90 - 8y$

Or, $9y = 90$

$\therefore \quad y = 10$, 3d;
From (2), $w = 18$, 1st;
" (3), $x = 22$, 2d;
" (5), $z = 40$, 4th, } *Ans.*

8. Let $x =$ A's distance,

$y =$ B's "

And $z =$ C's "

Then $x + y + z = 62$ (1)

$x = 4z + 2y$ (2)

And $2x + 3y = 17z$ (3)

Subtracting (2) from (1), $3y + 5z = 62$ (4)

From (3), $2x + 3y - 17z = 0$ (5)

Mult. (2) by 2, $2x - 4y - 8z = 0$ (6)

Subt. (6) from (5), $7y - 9z = 0$ (7)

Multiplying (4) by 7, $21y + 35z = 62 \times 7$

" (7) by 3, $21y - 27z = 0$

Subtracting, $62z = 62 \times 7$

$\therefore \quad = 7$ miles, C's distance;
From (7), $y = 9$ " B's "
" (2), $x = 46$ " A's " } *Ans.*

9. Let $4x$ = A's money,
$2y$ = B's "
And $3z$ = C's "
Then $4x + y = \$100$ (1)
$2y + z = \$100$ (2)
And $3z + x = \$100$ (3)
Mult. (3) by 4, $4x + 12z = 400$ (4)
Subt. (1) from (4), $12z - y = 300$ (5)
Mult. (5) by 2, $-2y + 24z = 600$ (6)
Adding (6) and (2), $25z = 700$
$\therefore z = \$28$
And $3z = \$84$, C's;
From (4), $4x = \$64$, A's;
" (2), $2y = \$72$, B's, } *Ans.*

GENERALIZATION.

Pages 124, 125.

1. $x = \frac{a}{b} = \frac{225}{75} = 3$ chickens, *Ans.*

2. $x = \frac{480}{16} = 30$ rods, *Ans.*

3. $x = \frac{576}{48} = 12$, *Ans.*

4. $x = \frac{a}{bc} = \frac{61320}{30 \times 40} = 51\frac{1}{10}$ years, C's age, *Ans.*

5. $x = \frac{504}{9 \times 8} = 7$ feet, *Ans.*

6. $x = \frac{62730}{41 \times 45} = 34$, *Ans.*

7. $g = \frac{s + d}{2} = \frac{392 + 18}{2} = \$205$;
$l = \frac{s - d}{2} = \frac{392 - 18}{2} = \$187$, } *Ans.*

Pages 126–127.

8. $g = \frac{1575 + 347}{2} = \$961$, A's part;
$l = \frac{1575 - 347}{2} = \$614$, B's " } *Ans.*

9. $g = \frac{2150 + 346}{2} = 1248$ votes;
$l = \frac{2150 - 346}{2} = 902$ " } *Ans.*

10. $x = \frac{ab}{a + b} = \frac{8 \times 12}{8 + 12} = \frac{96}{20} = 4\frac{4}{5}$ days, *Ans.*

11. $x = \frac{9 \times 15}{9 + 15} = \frac{135}{24} = 5\frac{5}{8}$ hours, *Ans.*

Pages 128, 129.

12. $x = \frac{40 \times 50}{40 + 50} = \frac{2000}{90} = 22\frac{2}{9}$ hours, *Ans.*

13. $p = br = 748 \times .09 = \$67.32$, *Ans.*

14. $p = 45385 \times .20 = 9077$, *Ans.*

15. $p = 2763 \times .375 = \$1036.12\frac{1}{2}$, *Ans.*

16. $p = 1587 \times .37 = 587.19$ bushels, *Ans.*

17. $r = \frac{p}{b} = \frac{336}{2700} = .12\frac{4}{9}$, *Ans.*

18. $r = \frac{25.2}{63} = .40$, *Ans.*

19. $r = \frac{291}{485} = .60$, *Ans.*

20. $b = \frac{p}{r} = \frac{750}{.25} = \$3000$, *Ans.*

Pages 130–133.

21. $b = \frac{750}{.02} = \$37500$, *Ans.*

22. $b = \frac{2500}{.08} = \$31250$, *Ans.*

23. $a = b(1+r) = 2500 \times 1.08 = \$2700$, B's;
$a = b(1-r) = 2500 \times .92 = \$2300$, C's, } *Ans.*

24. $a = b(1+r) = 4500 \times 1.25 = \$5625$, *Ans.*

25. $a = b(1-r) = 2750 \times .67 = 1842\frac{1}{2}$ acres, *Ans.*

26. $i = prt = 465 \times .06 \times 2 = \$55.80$, *Ans.*

27. $i = 1586 \times .08 \times 1\frac{1}{2} = \$190.32$, *Ans.*

28. $i = 3580 \times .07 \times 5 = \$1253$, *Ans.*

29. $a = p + prt = 364 + 364 \times .15 = \$418.60$, *Ans.*

30. $a = 4375 + 4375 \times .20 = \$5250$, *Ans.*

31. $a = 2863.60 + 2863.60 \times .35 = \$3865.86$, *Ans.*

32. $p = \frac{a}{1+rt} = \frac{1500}{1.12} = \$1339.29$, *Ans.*

33. $p = \frac{300}{1.35} = \$222.222$, *Ans.*

34. $t = \frac{a-p}{pr} = \frac{525}{3500 \times .06} = \frac{525}{210} = 2\frac{1}{2}$ yrs., *Ans.*

36. $t = ah = \frac{12}{11} \times 3 = \frac{36}{11} = 3:16\frac{4}{11}$ o'clock, P. M., *Ans.*

37. $t = \frac{12}{11} \times 6 = \frac{72}{11} = 6:32\frac{8}{11}$ o'clock, P. M., *Ans.*

38. $t = \frac{12}{11} \times 9 = \frac{108}{11} = 9:49\frac{1}{11}$ o'clock, P. M., *Ans.*

INVOLUTION.

Page 137.

2. $(abc)^2 = a^2b^2c^2$.
3. $(-abc)^2 = a^2b^2c^2$.
4. $(xyz)^3 = x^3y^3z^3$.
5. $(abc)^5 = a^5b^5c^5$.
6. $(2x^2y)^4 = 16x^8y^4$.
7. $(6a^3b^2)^3 = 216a^9b^6$.
8. $(5a^3b^2c)^4 = 625a^{12}b^8c^4$.
9. $(2a^2bc^2)^6 = 64a^{12}b^6c^{12}$.
10. $(abcd)^8 = a^8b^8c^8d^8$.
11. $(xyz)^n = x^ny^nz^n$.

Page 138.

12. The 5th power of $(a + b)^2 = (a + b)^{10}$.

13. The 2d power of $(a + b)^n = (a + b)^{2n}$.

14. The nth power of $(x - y)^m = (x - y)^{mn}$.

15. The nth power of $(x + y)^2 = (x + y)^{2n}$.

16. The 2d power of $(a^3 + b^3) = (a^3 + b^3)^2$.

17. The 3d power of $(a^3b^2h^4) = a^9b^6h^{12}$.

19. $\left(\frac{3ab^2}{2a}\right)^3 = \frac{27a^3b^6}{8a^3}$.

20. $\left(\frac{2a^2bc^3}{x^2y^3}\right)^4 = \frac{16a^8b^4c^{12}}{x^8y^{12}}$.

21. $\left(\frac{7a^nb^2}{3a^2b^n}\right)^2 = \frac{49a^{2n}b^4}{9a^4b^{2n}}$.

22. $\left(\frac{2}{a}\right)^m = \frac{2^m}{a^m}$.

23. $\left(\frac{a^mb^n}{xy^n}\right)^n = \frac{a^{mn}b^{n\times n}}{x^ny^{n\times n}}$, or $= \frac{a^{mn}b^{n^2}}{x^ny^{n^2}}$.

26. $(x + 2y + 2)^3 = x^3 + 6x^2y + 6x^2 + 12xy^2 + 24xy + 12x + 8y^3 + 24y^2 + 24y + 8$.

Page 142.

1. $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

2. $(a - b)^5 = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$.

3. $(c + d)^7 = c^7 + 7c^6d + 21c^5d^2 + 35c^4d^3 + 35c^3d^4 + 21c^2d^5 + 7cd^6 + d^7$.

4. $(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$.

5. $(x - y)^7 = x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$.

6. $(y + z)^{10} = y^{10} + 10y^9z + 45y^8z^2 + 120y^7z^3 + 210y^6z^4 + 252y^5z^5 + 210y^4z^6 + 120y^3z^7 + 45y^2z^8 + 10yz^9 + z^{10}$.

7. $(a - b)^9 = a^9 - 9a^8b + 36a^7b^2 - 84a^6b^3 + 126a^5b^4 - 126a^4b^5 + 84a^3b^6 - 36a^2b^7 + 9ab^8 - b^9$.

8. $(m + n)^{11} = m^{11} + 11m^{10}n + 55m^9n^2 + 165m^8n^3 + 330m^7n^4 + 462m^6n^5 + 462m^5n^6 + 330m^4n^7 + 165m^3n^8 + 55m^2n^9 + 11mn^{10} + n^{11}$.

9. $(x-y)^{12} = x^{12} - 12x^{11}y + 66x^{10}y^2 - 220x^9y^3 + 495x^8y^4 - 792x^7y^5 + 924x^6y^6 - 792x^5y^7 + 495x^4y^8 - 220x^3y^9 + 66x^2y^{10} - 12xy^{11} + y^{12}$.

10. $(a+b)^n = a^n + na^{n-1}b + n\frac{n-1}{2}a^{n-2}b^2 + n\frac{n-1}{2}\times\frac{n-2}{3}a^{n-3}b^3 + n\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{n-3}{4}a^{n-4}b^4$, etc.

13. $(x+1)^3 = x^3 + 3x^2 + 3x + 1$.

14. $(b-1)^4 = b^4 - 4b^3 + 6b^2 - 4b + 1$.

15. $(1-a)^5 = 1 - 5a + 10a^2 - 10a^3 + 5a^4 - a^5$.

16. $(1+a)^n = 1 + na + n\frac{n-1}{2}a^2 + n\frac{n-1}{2}\times\frac{n-2}{3}a^3 + \text{etc.}$

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17. $(x+y+z)^3 = x^3 + 3x^2y + 3x^2z + 3xy^2 + 6xyz + 3xz^2 + y^3 + 3y^2z + 3yz^2 + z^3$.

20. $(x+y+z)^2 = x^2+y^2+z^2+2xy+2xz+2yz$, or
$= x^2 + 2x(y+z) + y^2 + 2yz + z^2$, *Ans.*

21. $(a-b+c)^2 = a^2+b^2+c^2-2ab+2ac-2bc$, or
$= a^2 - 2a(b-c) + b^2 - 2bc + c^2$, *Ans.*

22. $(a+x+y+z)^2$
$= a^2+x^2+y^2+z^2+2ax+2ay+2az+2xy+2xz+2yz$, or
$= a^2 + 2a(x+y+z) + x^2 + 2x(y+z) + y^2 + 2yz + z^2$, *Ans.*

Page 144.

24. $\left(a+\frac{2}{3}\right)^2 = \left(\frac{3a+2}{3}\right)^2 = \frac{9a^2+12a+4}{9}$, *Ans.*

25. $\left(a-\frac{c}{2}\right)^2 = \left(\frac{2a-c}{2}\right)^2 = \frac{4a^2-4ac+c^2}{4}$, *Ans.*

26. $\left(-\frac{6}{7}+2abc\right)^2=\left(\frac{-6+14abc}{7}\right)^2$

$$=\frac{36-168abc+196a^2b^2c^2}{49}, \textit{ Ans.}$$

27. $\left(-\frac{b}{m}+3xy\right)^2=\left(\frac{-b+3mxy}{m}\right)^2$

$$=\frac{b^2-6bmxy+9m^2x^2y^2}{m^2}, \textit{ Ans.}$$

MULTIPLICATION AND DIVISION OF POWERS.

Page 145.

3. $a^6\times a^2=a^8$.
4. $x^{-5}\times x^{-3}=x^{-8}$.
5. $b^{-7}\times b^4=b^{-3}$.
6. $a^m\times a^n=a^{m+n}$.
7. $a^{-4}b\times a^{-3}b^4=a^{-7}b^5$.
8. $a^{-4}cd\times a^7c^3d^2=a^3c^4d^3$.
9. $b^4c^{-6}y^{-2}\times b^{-2}c^8y^4=b^2c^2y^2$.
10. $a^6y^{-4}z^3\times a^{-3}y^{-5}z^4=a^3y^{-9}z^7$.

Page 146.

12. $a^8\div a^{-3}=a^{11}$.
13. $x^{-8}\div x^3=x^{-11}$.
14. $b^4\div b^2=b^2$.
15. $c^{-5}\div c^{-3}=c^{-2}$.
16. $x^4yz^2\div x^{-3}y^4z^{-3}=x^7y^{-3}z^5$.
17. $12a^3b^{-2}c\div 3a^2b^{-3}c^5=4abc^{-4}$.
18. $6x^4y^2z^2\div 2x^{-2}yz^2=3x^6y$.
19. $60a^3b^4c^5\div 5a^{-2}b^4c^{-3}=12a^5c^8$.
24. $\frac{ax^{-3}}{y}=\frac{a}{x^3y}$, *Ans.*
25. $\frac{a}{by^4}=\frac{ay^{-4}}{b}$, *Ans.*
26. $\frac{ad^{-5}}{x^2}=\frac{a}{d^5x^2}$, *Ans.*
27. $\frac{b}{ax^n}=\frac{bx^{-n}}{a}$, *Ans.*

EVOLUTION.

Pages 148, 149.

13. $a^{\frac{4}{5}}$.
14. $x^{\frac{5}{4}}$.
15. $y^{\frac{12}{8}}$.
16. $a^{\frac{1}{4}} = a^{.25}$, *Ans.*
17. $a^{\frac{1}{5}} = a^{.2}$, *Ans.*
18. $a^{\frac{3}{5}} = a^{.6}$, *Ans.*
19. $b^{\frac{4}{5}} = b^{.8}$, *Ans.*
20. $x^{\frac{7}{5}} = x^{1.4}$, *Ans.*
21. $y^{\frac{3}{4}} = y^{.75}$, *Ans.*

Page 151.

3. $\sqrt{a^6} = a^3$, *Ans.*
4. $\sqrt[4]{a^1 \text{ or } a} = a^{\frac{1}{4}}$, *Ans.*
5. $\sqrt[3]{4xy} = 4^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{1}{3}}$, *Ans.*
6. $\sqrt[3]{8a^5b^6} = 2a^{\frac{5}{3}}b^2$, *Ans.*
7. $\sqrt[3]{27abc} = 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$, *Ans.*
8. $\sqrt[4]{16a^{4m}} = 2a^m$, *Ans.*
9. $\sqrt[5]{3a^3x^{10}} = 3^{\frac{1}{5}}a^{\frac{3}{5}}x^2$, *Ans.*
10. $\sqrt{36a^4b^2} = 6a^2b$, *Ans.*
11. $\sqrt[3]{2x^2y^2} = 2^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{2}{3}}$, *Ans.*
12. $\sqrt{64a^3b^6} = 8a^{\frac{3}{2}}b^3$, *Ans.*
13. $\sqrt[5]{13xy} = (13)^{\frac{1}{5}}x^{\frac{1}{5}}y^{\frac{1}{5}}$, *Ans.*
14. $\sqrt{49x^4y^6} = 7x^2y^3$, *Ans.*
15. $\sqrt[3]{27a^2b^2} = 3a^{\frac{2}{3}}b^{\frac{2}{3}}$, *Ans.*
16. $\sqrt{\frac{49x^4}{64y^2}} = \frac{7x^2}{8y}$, *Ans.*

Page 152.

2. $x + 2$.
3. $a - 1$.
4. $1 + x$.
5. $x + \frac{2}{3}$.
6. $a - \frac{1}{2}$.
7. $x + \frac{b}{2}$.

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9. $x^2 + 2xy + y^2 + 2xz + 2yz + z^2$ ($x + y + z$, *Ans.*

x^2

$2x + y$) $2xy + y^2$

$2x + 2y + z$) $2xz + 2yz + z^2$

10. $a^2 - 4ab + 4b^2 + 2a - 4b + 1$ ($a - 2b + 1$, *Ans.*

a^2

$2a - 2b$) $-4ab + 4b^2$

$2a - 4b + 1$) $2a - 4b + 1$

11. $a^4+4a^2b+4b^2-4a^2-8b+4 \;(\; a^2+2b-2,$ *Ans.*

$$\begin{array}{r|l} a^4 & \\ \hline 2a^2+2b & 4a^2b+4b^2 \\ & 2a^2+4b-2 \\ \hline & -4a^2-8b+4 \end{array}$$

12. $1-4b^2+4b^4+2x-4b^2x+x^2 \;(\; 1-2b^2+x,$ *Ans.*

$$\begin{array}{r|l} 1 & \\ \hline 2-2b^2 & -4b^2+4b^4 \\ & 2-4b^2+x \\ \hline & 2x-4b^2x+x^2 \end{array}$$

13. $4a^4-16a^3+24a^2-16a+4 \;(\; 2a^2-4a+2,$ *Ans.*

$$\begin{array}{r|l} 4a^4 & \\ \hline 4a^2-4a & -16a^3+24a^2 \\ & -16a^3+16a^2 \\ \hline 4a^2-8a+2 & 8a^2-16a+4 \\ & 8a^2-16a+4 \\ \hline \end{array}$$

14. $a^2-ab+\frac{b^2}{4} \;\Big|\; a-\frac{b}{2},$ *Ans.*

$$\begin{array}{r|l} a^2 & \\ \hline 2a-\frac{b}{2} & -ab+\frac{b^2}{4} \\ & -ab+\frac{b^2}{4} \\ \hline & \end{array}$$

15. $\frac{x^2}{y^2}-2+\frac{y^2}{x^2} \;\Big|\; \frac{x}{y}-\frac{y}{x},$ *Ans.*

$$\begin{array}{r|l} \frac{x^2}{y^2} & \\ \hline \frac{2x}{y}-\frac{y}{x} & -2+\frac{y^2}{x^2} \\ & -2+\frac{y^2}{x^2} \\ \hline & \end{array}$$

NOTE. $-2+\frac{2x}{y}=-2\times\frac{y}{2x}=-\frac{y}{x}$, second term of root;

Also, $\frac{2x}{y}\times-\frac{y}{x}=-2$; and $-\frac{y}{x}\times-\frac{y}{x}=\frac{y^2}{x^2}$.

REDUCTION OF RADICALS.

Case I, Page 156.

6. $a\sqrt{b}$.
7. $2a\sqrt{2b}$.
8. $6\sqrt{xy}$.
9. $6\sqrt[3]{3}$.
10. $15\sqrt[3]{5}$.
11. $6\sqrt{36 \times 7a^2b}$
$= 36a\sqrt{7b}$, *Ans.*
12. $3a\sqrt[3]{2c}$.
13. $21a\sqrt{1 - 3b}$.
14. $4x\sqrt[3]{y}$.
15. $3a\sqrt[4]{b}$.
16. $6a\sqrt{13c}$.
17. $\sqrt{1584a^2} = \sqrt{144a^2 \times 11}$
$= 12a\sqrt{11}$, *Ans.*

Case II, Page 157.

2. $2a^3b = \sqrt[3]{8a^9b^3}$, *Ans.*
3. $\sqrt{(2a + b)^2}$.
4. $\sqrt{(a - 2b)^2}$.
5. $\sqrt{9a^2b}$.
6. $\sqrt[3]{8a^7b}$.
7. $\sqrt[4]{16x^2y^{\frac{4}{3}}z}$.
8. $\sqrt[3]{\frac{a^3b^3c^3}{27}}$.
9. $\sqrt[3]{27\,(a - b)^3}$.
10. $\sqrt[3]{a^6}$.
11. $\sqrt[4]{a^{12}c^8}$.
12. $\sqrt{(a - b)^2}$.
13. $\sqrt[n]{a^{mn}}$.

Case III, Page 158.

2. $a^{\frac{1}{3}} = a^{\frac{2}{6}} = (a^2)^{\frac{1}{6}}$;
$(bc)^{\frac{2}{3}} = (bc)^{\frac{4}{6}} = (b^4c^4)^{\frac{1}{6}}$, } *Ans.*

3. $3^{\frac{1}{2}} = 3^{\frac{2}{4}} = 9^{\frac{1}{4}}$; } *Ans.*
$5^{\frac{2}{4}} = (125)^{\frac{1}{4}}$,

4. $a^{\frac{1}{3}} = a^{\frac{2}{6}} = (a^2)^{\frac{1}{6}}$; } *Ans.*
$6^{\frac{2}{3}} = 6^{\frac{4}{6}} = (1296)^{\frac{1}{6}}$,

5. $\sqrt{5} = \sqrt[12]{5^6} = \sqrt[12]{15625}$; }
$\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$; } *Ans.*
$\sqrt[4]{2} = \sqrt[12]{2^3} = \sqrt[12]{8}$, }

6. $\sqrt[6]{4x^8}$ and $\sqrt[6]{125x^9}$.

7. $\sqrt[6]{64a^9}$ and $\sqrt[6]{4a^4}$.

8. $\sqrt[mn]{a^n}$ and $\sqrt[mn]{b^m}$.

9. $(b^n)^{\frac{1}{2n}}$ and $(c^2)^{\frac{1}{2n}}$.

10. $\sqrt[6]{(a+b)^3}$ and $\sqrt[6]{(a-b)^2}$.

11. $\sqrt[12]{(x-y)^8}$ and $\sqrt[12]{(x+y)^9}$.

Case IV, Page 159.

3. $(3^{\frac{2}{3}})^{\frac{1}{2}}$ and $(4^2)^{\frac{1}{2}}$.

4. $(a^{10})^{\frac{1}{2}}$ and $(b^{15})^{\frac{1}{2}}$.

5. $3 \div \frac{3}{4} = 3 \times \frac{4}{3} = 4$; $\quad \frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$.

Hence, $(a^4)^{\frac{1}{2}}$ and $(b^{\frac{1}{3}})^{\frac{1}{2}}$. *Ans.*

6. $(a^{\frac{1}{3}})^{\frac{1}{8}}$ and $(b^{1\frac{1}{2}})^{\frac{1}{8}}$.

7. $(a^{\frac{n}{m}})^{\frac{1}{n}}$ and $(b^{\frac{2n}{m}})^{\frac{1}{n}}$.

ADDITION OF RADICALS.

Page 160.

4. $\sqrt{12} = 2\sqrt{3}$
$\sqrt{27} = 3\sqrt{3}$
$5\sqrt{3}$, *Ans.*

5. $\sqrt{20} = 2\sqrt{5}$
$\sqrt{48} = 4\sqrt{3}$
$2\sqrt{5}+4\sqrt{3}$, *Ans.*

6. $2\sqrt{b^3} = 2b\sqrt{b}$
$3\sqrt{a^2b} = 3a\sqrt{b}$
$(2b + 3a)\sqrt{b}$, *Ans.*

7. $a\sqrt{3a^3b} = a^2\sqrt{3ab}$
$c\sqrt{27ab} = 3c\sqrt{3ab}$
$(a^2 + 3c)\sqrt{3ab}$, *Ans.*

8. $3\sqrt{18ax^2} = 9x\sqrt{2a}$
$2\sqrt{32a^3} = 8a\sqrt{2a}$
$(9x + 8a)\sqrt{2a}$, *Ans.*

9. $3\sqrt[3]{54} = 9\sqrt[3]{2}$
$4\sqrt[3]{128} = 16\sqrt[3]{2}$
$25\sqrt[3]{2}$, *Ans.*

10. $7\sqrt{243} = 7\sqrt{81 \times 3} = 63\sqrt{3}$
$5\sqrt{363} = 5\sqrt{121 \times 3} = 55\sqrt{3}$
$118\sqrt{3}$, *Ans.*

11. $a\sqrt{81b} = 9a\sqrt{b}$
$3a\sqrt{49b} = 21a\sqrt{b}$
$30a\sqrt{b}$, *Ans.*

12. $b\sqrt{25x^2c} = 5bx\sqrt{c}$
$\sqrt{36x^4c} = 6x^2\sqrt{c}$
$(5bx + 6x^2)\sqrt{c}$, *Ans.*

13. $4\sqrt[3]{x^6y} = 4x^2\sqrt[3]{y}$
$5\sqrt{x^3y} = 5x\sqrt{xy}$
$4x^2\sqrt[3]{y} + 5x\sqrt{xy}$, *Ans.*

SUBTRACTION OF RADICALS.

Page 161.

2. $4\sqrt{112} = 16\sqrt{7}$
$\sqrt{448} = 8\sqrt{7}$
$8\sqrt{7}$, *Ans.*

3. $\sqrt{480} = 4\sqrt{30}$
$4\sqrt{63} = 12\sqrt{7}$
$4\sqrt{30} - 12\sqrt{7}$, *Ans.*

4. $4\sqrt{320} = 32\sqrt{5}$
$-5\sqrt{80} = -20\sqrt{5}$
$52\sqrt{5}$, *Ans.*

5. $3\sqrt{49ax^2} = 21x\sqrt{ax}$
$2\sqrt{25ax} = 10\sqrt{ax}$
$(21x - 10)\sqrt{ax}$, *Ans.*

6. $5\sqrt[3]{a+b} - 3\sqrt[3]{a+b} = 2\sqrt[3]{a+b}$, *Ans.*

7. $3\sqrt[3]{b} - (-4\sqrt[3]{b}) = 7\sqrt[3]{b}$, *Ans.*

8. $3\sqrt[3]{250b^4x} = 15b\sqrt[3]{2bx}$
$2\sqrt[3]{54b^4x} = 6b\sqrt[3]{2bx}$
$9b\sqrt[3]{2bx}$, *Ans.*

9. $-a^{\frac{1}{2}} - (-2a^{\frac{1}{2}}) = a^{\frac{1}{2}}$, *Ans.*

10. $5\sqrt{\frac{2}{4}} = \frac{5}{2}\sqrt{3} = \frac{15}{6}\sqrt{3}$
$2\sqrt{\frac{1}{3}} = \frac{2}{3}\sqrt{3} = \frac{4}{6}\sqrt{3}$
$1\frac{5}{6}\sqrt{3}$, *Ans.*

MULTIPLICATION OF RADICALS.

Page 162.

4. $5\sqrt{18} = 15\sqrt{2}$
$3\sqrt{20} = 6\sqrt{5}$
$90\sqrt{10}$, *Ans.*

5. abx.

6. $\sqrt{a^2 - b^2}$.

7. $\sqrt{acxy}$.

8. $a^{\frac{3}{2}} = \sqrt{a^3}$; $\sqrt{a^3} \times \sqrt{c} = a\sqrt{ac}$, *Ans.*

20. $a^{\frac{1}{m}} = a^{\frac{n}{mn}} = \sqrt[mn]{a^n}$
$x^{\frac{1}{n}} = x^{\frac{m}{mn}} = \sqrt[mn]{x^m}$
$\sqrt[mn]{a^n x^m}$, *Ans.*

11. $7\sqrt[3]{4} \times 3\sqrt[3]{4} = 21\sqrt[3]{16} = 42\sqrt[3]{2}$, *Ans.*

12. $12a$.

13. 6.

14. $4ax$.

16. $2\sqrt{\frac{5}{8}} \times 2\sqrt{\frac{1}{2}} = 4\sqrt{\frac{5}{16}} = \sqrt{5}$, *Ans.*

17. $4\sqrt{\frac{5}{9}} \times 3\sqrt{\frac{1}{4}} = 12\sqrt{\frac{5}{36}} = 2\sqrt{5}$, *Ans.*

18. $(m+n)^{\frac{3}{4}} \times (m+n)^{\frac{3}{4}} = (m+n)^{\frac{6}{4}}$;
$(m+n)^{\frac{6}{4}} = (m+n)\sqrt[4]{m+n}$, *Ans.*

19. $\sqrt{\frac{9ad}{2b}} \times \sqrt{\frac{2ab}{3c}} = a\sqrt{\frac{3d}{c}}$.

DIVISION OF RADICALS.

Page 164.

4. $\sqrt{12a^3c} \div \sqrt{4c} = \sqrt{3a^3}$
or, $= a\sqrt{3a}$. (Art. 306.)

5. $3\sqrt{bx}$.

6. $(a^2+x)^{\frac{1}{3}}$.

7. $12\,(ay)^{\frac{1}{2}}$.

8. $3b\sqrt{x}$.

9. $\frac{15a}{2}\sqrt{b}$.

10. $2a\sqrt{x}$.

11. $\frac{2}{n} - \frac{1}{n} = \frac{1}{n}$;
$(a+b)^{\frac{1}{n}}$, *Ans.*

12. $3\sqrt{25x^3} = 15x\sqrt{x}$, *Ans.*

13. $\sqrt[3]{x-y}$. (Art. 128.)

14. $8\sqrt{8} = 8\sqrt{4 \times 2} = 16\sqrt{2}$, *Ans.*

15. $2\sqrt{256} = 2 \times 16 = 32$, *Ans.*

INVOLUTION OF RADICALS.

Page 164.

3. $a^{\frac{3}{4}}$.

4. $(3\sqrt{2x})^2 = 9 \times 2x = 18x$, *Ans.*

5. $8a$.

6. $\left(\frac{x}{2}\sqrt{2x}\right)^3 = \frac{x^3}{8} \times 2x\sqrt{2x} = \frac{x^4}{4}\sqrt{2x}$, *Ans.*

7. $\left(4\sqrt{\frac{ax^2}{4}}\right)^3 = (2x\sqrt{a})^3 = 8ax^3\sqrt{a}$, *Ans.*

8. $\left(3\sqrt{\frac{b}{3}}\right)^4 = \frac{81b^2}{9} = 9b^2$, *Ans.*

9. $a^2 + 2a\sqrt{y} + y$, (Art. 266).

EVOLUTION OF RADICALS.

Page 165.

2. $3\sqrt[6]{a^2} = 3\sqrt[3]{a}$, *Ans.*

3. $2\sqrt[6]{3x}$.

4. $(3\sqrt{xy})^{\frac{1}{3}} = (\sqrt{9xy})^{\frac{1}{3}} = \sqrt[6]{9xy}$, *Ans.*

5. $(2b\sqrt{2b})^{\frac{1}{3}} = (\sqrt{8b^3})^{\frac{1}{3}} = \sqrt[6]{8b^3}$, *Ans.*

6. $\sqrt[6]{a^2bc}$.

7. $\left(\frac{4}{9}\sqrt[3]{\frac{4}{9}}\right)^{\frac{1}{4}} = \left(\sqrt[3]{\frac{4^3}{9^3} \times \frac{4}{9}}\right)^{\frac{1}{4}} = \sqrt[12]{\frac{256}{6561}} = \sqrt[3]{\frac{4}{9}}$, *Ans.*

8. $\sqrt[4]{a^{\frac{3}{2}}c^{\frac{3}{2}}} = ac^{\frac{1}{2}}$, *Ans.*

9. $(128\sqrt[7]{a})^{\frac{1}{7}} = (2^7a^{\frac{1}{7}})^{\frac{1}{7}} = 2a^{\frac{1}{49}}$, *Ans.*

10. $\sqrt[4]{a^{\frac{1}{2}}b^{\frac{1}{2}}} = a^{\frac{1}{8}}b^{\frac{1}{8}}$, *Ans.*

11. $(4a^2\sqrt{2a})^{\frac{1}{3}} = (\sqrt{32a^5})^{\frac{1}{3}} = \sqrt[10]{32a^5} = \sqrt{2a}$, *Ans.*

12. $a^{\frac{1}{n}}b^{\frac{1}{2n}}c^{\frac{1}{4n}}$.

REDUCING A RADICAL TO A RATIONAL QUANTITY.

Case I, Page 166.

4. $\frac{3}{3} - \frac{2}{3} = \frac{1}{3}$; $a^{\frac{1}{3}}$, *Ans.*

5. $a^{\frac{2}{3}}c^{\frac{1}{3}} \times a^{\frac{1}{3}}c^{\frac{2}{3}} = ac$; hence $a^{\frac{1}{3}}c^{\frac{2}{3}}$, *Ans.*

6. $(a+b)^{\frac{2}{3}} \times (a+b)^{\frac{1}{3}} = (a+b)$; hence $(a+b)^{\frac{2}{3}}$, *Ans.*

7. $\sqrt{a^3b^2c} = ab\sqrt{ac}$

$ab\sqrt{ac} \times \sqrt{ac} = a^2bc$. Hence $\sqrt{ac}$, *Ans.*

8. $\sqrt[3]{(x+y)^2} \times \sqrt[3]{x+y} = x+y$. Hence $\sqrt[3]{x+y}$, *Ans.*

9. $\sqrt[4]{(a+b)^3} \times \sqrt{a+b} = a+b$. Hence $\sqrt[4]{a+b}$, *Ans.*

10. $\sqrt{a+b+c} \times \sqrt{a+b+c} = a+b+c$. Hence $\sqrt{a+b+c}$, *Ans.*

Case II, Page 167.

3. $x - 4\sqrt{9}$.

4. $(\sqrt{9} - \sqrt{6})(\sqrt{9} + \sqrt{6}) = 9 - 6 = 3$, *Ans.*

5. $\sqrt{7} - \sqrt{a}$.

6. $(6 - \sqrt{5})(6 + \sqrt{5}) = 36 - 5 = 31$, *Ans.*

7. $\sqrt{3a} + \sqrt{3b}$.

8. $(\sqrt{a} - \sqrt{5})(\sqrt{a} + \sqrt{5}) = a - 5$, *Ans.*

9. $3\sqrt{a} - \sqrt{8}$.

10. $4\sqrt{2a} + 5\sqrt{b}$.

Case III, Page 168.

4. $\dfrac{c}{\sqrt[3]{x}} \times \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{c\sqrt[3]{x^2}}{x}$, *Ans.*

5. $\dfrac{a}{\sqrt[3]{c}} \times \dfrac{\sqrt[3]{c^2}}{\sqrt[3]{c^2}} = \dfrac{a\sqrt[3]{c^2}}{c}$, *Ans.*

6. $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \times \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{x+2\sqrt{xy}+y}{x-y}$, *Ans.*

7. $\dfrac{x}{\sqrt{a}-\sqrt{c}} \times \dfrac{\sqrt{a}+\sqrt{c}}{\sqrt{a}+\sqrt{c}} = \dfrac{x(\sqrt{a}+\sqrt{c})}{a-c}$, *Ans.*

8. $\dfrac{1}{1+\sqrt{3}} \times \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{1-\sqrt{3}}{1-3} = \dfrac{1-\sqrt{3}}{-2}$

$$= \frac{\sqrt{3}-1}{2}, \textit{ Ans.}$$

NOTE.—Changing the signs of both terms of the fraction does not alter its value. (Art. 166.)

9. $\dfrac{\sqrt{3}}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}} = \dfrac{3\sqrt{3}+3}{9-3} = \dfrac{3(\sqrt{3}+1)}{6}$

$$= \frac{\sqrt{3}+1}{2}, \textit{ Ans.}$$

RADICAL EQUATIONS.

Page 169.

4. Given $a + \sqrt[n]{x} + c = d$

Transposing, $\sqrt[n]{x} = d - a - c$

Involving, $x = (d-a-c)^n$, *Ans.*

5. Given $\sqrt[3]{x+2} = 3$

Involving, $x + 2 = 27$

$\therefore \quad x = 25$, *An*

6. Given $3\sqrt{x-4} + 5 = 7\frac{1}{2}$

Transposing, $3\sqrt{x-4} = 2\frac{1}{2} = \frac{5}{2}$

Dividing, $\sqrt{x-4} = \frac{5}{6}$

Involving, $x - 4 = \frac{25}{36}$

$\therefore \quad x = 4\frac{25}{36}$, *Ans.*

7. Given $3\sqrt{\dfrac{x}{4}} = 24$

Dividing, $\sqrt{\dfrac{x}{4}} = 8$

Involving, $\dfrac{x}{4} = 64$

$\therefore \quad x = 256$, *Ans.*

8. Given $\sqrt[3]{2x+3}-6=13$
Involving, $2x+3-6=2197$
Transposing, $2x=2200$
$\therefore \quad x=1100$, *Ans.*

9. Given $\sqrt[3]{x-4}=3$
Involving, $x-4=27$
$\therefore \quad x=31$, *Ans.*

10. Given $2\sqrt[4]{x-5}=4$
Dividing, $\sqrt[4]{x-5}=2$
Involving, $x-5=16$
$\therefore \quad x=21$, *Ans.*

11. Given $5\sqrt{\frac{x}{7}}=30$

Involving, etc., $\frac{x}{7}=36$
$\therefore \quad x=252$, *Ans.*

Page 170.

12. Given $\frac{a+\sqrt{2ax+b}}{b}=b^2$

Multiplying, etc., $\sqrt{2ax+b}=b^3-a$
Involving, $2ax+b=b^6-2ab^3+a^2$
Transposing, $2ax=b^6-2ab^3+a^2-b$
$\therefore \quad x=\frac{b^6-2ab^3+a^2-b}{2a}$, *Ans.*

14. Given $\frac{y-ay}{\sqrt{y}}=\frac{\sqrt{y}}{y}$.

Clearing, etc., $y(y-ay)=y$
Dividing by y, $y-ay=1$
Factoring, $y(1-a)=1$
Dividing, $y=\frac{1}{1-a}$, *Ans.*

15. Given $x + \sqrt{a^2 + x^2} = \frac{2a^2}{\sqrt{a^2 + x^2}}$

Clearing of frac., $x\sqrt{a^2+x^2}+a^2+x^2 = 2a^2$

Transposing, $x\sqrt{a^2 + x^2} = a^2 - x^2$

Involving, $a^2x^2 + x^4 = a^4 - 2a^2x^2 + x^4$

Uniting, $3a^2x^2 = a^4$

Dividing by a^2, $3x^2 = a^2$

$\therefore \quad x^2 = \frac{a^2}{3}$

Extracting root, $x = a\sqrt{\frac{1}{3}}$, *Ans.*

17. Given $\sqrt{x + 12} = 2 + \sqrt{x}$

Involving, $x + 12 = 4 + 4\sqrt{x} + x$

Transposing, $4\sqrt{x} = 8$

Dividing, $\sqrt{x} = 2$

Squaring, $x = 4$, *Ans.*

18. Given $\sqrt{5} \times \sqrt{x + 2} = 2 + \sqrt{5x}$

Then $\sqrt{5x + 10} = 2 + \sqrt{5x}$

Squaring, $5x + 10 = 4 + 4\sqrt{5x} + 5x$

Uniting, $4\sqrt{5x} = 6$

Dividing, $\sqrt{5x} = \frac{3}{2}$

Involving, $5x = \frac{9}{4}$

$\therefore \quad x = \frac{9}{20}$, *Ans.*

19. Given $\frac{\sqrt{x}}{x} = \frac{x - ax}{\sqrt{x}}$

Clearing of fractions, $x = x(x - ax)$

Dividing by x, $(1 - a)x = 1$

$\therefore \quad x = \frac{1}{1 - a}$, *Ans.*

PURE QUADRATICS.

Page 173.

2. Given $3x^2 - 5 = 70$
Transposing, etc., $x^2 = 25$
Extracting root, $\therefore \quad x = \pm 5.$

3. Given $9x^2 + 8 = 3x^2 + 62$
Transposing, etc., $x^2 = 9$
$\therefore \quad x = \pm 3$, *Ans.*

4. Given $5x^2 + 9 = 2x^2 + 57$
Transposing, etc., $x^2 = 16$
$\therefore \quad x = \pm 4$, *Ans.*

5. Given $6x^2 + 5 = 4x^2 + 55$
Transposing, etc., $x^2 = 25$
$\therefore \quad x = \pm 5$, *Ans.*

6. Given $\frac{5x^2}{4} + 35 = 3x^2 + 7$
Multiplying by 4, $5x^2 + 140 = 12x^2 + 28$
Transposing, etc., $x^2 = 16$
$\therefore \quad x = \pm 4$, *Ans.*

7. Given $\frac{2x^2 + 8}{10} = \frac{x^2 - 6}{10} + 5$
Multiplying by 10, $2x^2 + 8 = x^2 - 6 + 50$
Transposing, etc., $x^2 = 36$
$\therefore \quad x = \pm 6$, *Ans.*

8. Given $\frac{x}{4} = \frac{x}{2} - \frac{4}{x}$
Multiplying by $4x$, $x^2 = 2x^2 - 16$
Transposing, $x^2 = 16$
$\therefore \quad x = \pm 4$, *Ans.*

9. Given $\frac{x}{2} + \frac{2}{x} = \frac{x}{3} + \frac{3}{x}$

Multiplying by $6x$, $3x^2 + 12 = 2x^2 + 18$

Transposing, $x^2 = 6$

$\therefore \quad x = \pm\sqrt{6}$, *Ans.*

10. Given $2x^2 + 12 = 3x^2 - 37$

Transposing, $x^2 = 49$

$\therefore \quad x = \pm 7$, *Ans.*

11. Given $7x^2 - 7 = 3x^2 + 9$

Transposing, etc., $x^2 = 4$

$\therefore \quad x = \pm 2$, *Ans.*

12. Given $a^2x^2 = a^4$

Dividing by a^2, $x^2 = a^2$

$\therefore \quad x = \pm a$, *Ans.*

13. Given $(x + 2)^2 = 4x + 5$

Or, $x^2 + 4x + 4 = 4x + 5$

Transposing, $x^2 = 1$

$\therefore \quad x = \pm 1$, *Ans.*

14. Given $x^2 - 1 = \frac{6x^2 - 12}{4}$

Multiplying by 4, $4x^2 - 4 = 6x^2 - 12$

Transposing, etc., $x^2 = 4$

$\therefore \quad x = \pm 2$, *Ans.*

15. Given $\frac{x(2x + 9)}{30} = \frac{3x + 6}{10}$

Multiplying by 30, $2x^2 + 9x = 9x + 18$

Transposing, etc., $x^2 = 9$

$\therefore \quad x = \pm 3$, *Ans.*

16. Given $\frac{5}{4 - x} + \frac{5}{4 + x} = \frac{8}{3}$

Clearing of fractions,

$60 + 15x + 60 - 15x = 128 - 8x^2$

Transposing, etc., $x^2 = 1$

$\therefore \quad x = \pm 1$, *Ans.*

Page 173—Continued.

17. Given $\frac{ax^2(a-2)}{1+x} = 1-x$

Clearing of frac., $a^2x^2 - 2ax^2 = 1 - x^2$

Transposing, $a^2x^2 - 2ax^2 + x^2 = 1$

Factoring, $(a^2 - 2a + 1)x^2 = 1$

Dividing, $x^2 = \frac{1}{a^2 - 2a + 1}$

$\therefore \quad x = \pm\frac{1}{a-1}$, *Ans.*

19. Given $2\sqrt{x^2 - 5} = \frac{4x}{3}$

Dividing by 2, and squaring,

$x^2 - 5 = \frac{4x^2}{9}$

Multiplying by 9, $9x^2 - 45 = 4x^2$

Transposing, etc., $x^2 = 9$

$\therefore \quad x = \pm 3$, *Ans.*

20. Given $2\sqrt{x^2 - 4} = 4\sqrt{a^2 - 1}$

Dividing by 2, and sq., $x^2 - 4 = 4a^2 - 4$

Transposing, $x^2 = 4a^2$

$\therefore \quad x = \pm 2a$, *Ans.*

21. Given $\sqrt{x+c} = \frac{d}{\sqrt{x-c}}$

Mult. by denominator, $x^2 - c^2 = d^2$

Transposing, $x^2 = c^2 + d^2$

$\therefore \quad x = \pm\sqrt{c^2+d^2}$, *Ans.*

22. Given $\sqrt{\frac{5x^2 - 1}{x}} = \sqrt{x}$

Squaring, etc., $5x^2 - 1 = x^2$

Transposing, etc., $x^2 = \frac{1}{4}$

$\therefore \quad x = \pm\frac{1}{2}$, *Ans.*

23. Given $\dfrac{b}{\sqrt{x-a}} = \sqrt{x+a}$

Squaring, etc., $b^2 = x^2 - a^2$

Transposing, $x^2 = a^2 + b^2$

$\therefore \quad x = \pm\sqrt{a^2+b^2}$, *Ans.*

24. Given $\dfrac{24}{\sqrt{x+10}} = \sqrt{x-10}$

Clearing of fractions, $24 = \sqrt{x^2 - 100}$

Squaring, $576 = x^2 - 100$

Transposing, $x^2 = 676$

$\therefore \quad x = \pm 26$, *Ans.*

PROBLEMS.

Page 174.

1. Let $x =$ the number.

Then $\frac{x}{3} \times \frac{x}{4} = 108$

Clearing of fractions, $x^2 = 1296$

$\therefore \quad x = \pm 36$, *Ans.*

2. Let $x =$ the number.

Then $25 - \frac{x^2}{4} = 9$

Multiplying by 4, $x^2 = 64$

$\therefore \quad x = \pm 8$, *Ans.*

3. Let $x =$ No. of rods on one side.

Then $x^2 = 1600$ sq. rds., area. (Ax. 10.)

$\therefore \quad x = 40$ rods, *Ans.*

NOTE.—It is advisable for the pupil to represent the area by a diagram, in this and like problems.

4. Let $x =$ length of side of square.

Then $x^2 = 50 \times 18 = 900$ rods, area.

$\therefore \quad x = 30$ rods, *Ans.*

5. Let $2x =$ one,
Then $5x =$ the other.
By conditions, $10x^2 = 360$
Dividing by 10, $x^2 = 36$
$\therefore \quad x = \pm 6$
And $\left.\begin{aligned} 2x &= 12; \\ 5x &= 30, \end{aligned}\right\}$ *Ans.*

6. Let $x =$ No. of dollars.
Then $x^2 - 7 = 29$
Transposing, $x^2 = 36$
$\therefore \quad x =$ \$6, *Ans.*

7. Let $x =$ number.
Then $\frac{x}{8} \times \frac{x}{5} \times \frac{1}{16} = 10$
Clearing of fractions, $x^2 = 6400$
$\therefore \quad x = 80$, *Ans.*

8. Let $x =$ less,
Then $4x =$ greater.
And $4x^2 = 900$
Dividing by 4, $x^2 = 225$
$\therefore \quad x = 15;$
And $4x = 60,$ $\left.\right\}$ *Ans.*

9. Let $x =$ number of yards.
Then $40\frac{1}{2} \div x =$ price.
$40\frac{1}{2} \div x = \frac{81}{2x}$
And $\frac{81}{2x} : x :: 3 : 54$
Mult., etc., $x^2 = 729$. (Art. 378.)
$\therefore \quad x = 27$ yards;
And $\frac{81}{2x} =$ \$1.50, price, $\left.\right\}$ *Ans.*

10. Let x = number.

Then $\frac{3x^2}{4} - 12 = 180$

Transposing, etc., $x^2 = 256$

$\therefore \quad x = \pm 16$, *Ans.*

11. Let x = length of side.

Then x^2 = area of bottom.

$6x^2$ = capacity in cubic feet.

By conditions, $6x^2 = \frac{266112 \times 231}{1728}$

Cancelling, $x^2 = \frac{\overset{77}{\cancel{266112}} \times \overset{77}{\cancel{231}}}{\cancel{2}, \cancel{6} \times \cancel{1728}}$

$x^2 = (77)^2$, and $x = 77$ feet, *Ans.*

12. Let x = number.

Then $(x + 10)(x - 10) = 156$

Or, $x^2 - 100 = 156$

Transposing, $x^2 = 256$

$\therefore \quad x = \pm 16$, *Ans.*

AFFECTED QUADRATICS.

First Method, Page 178.

6. Given $3x^2 - 24x = -36$

Dividing, $x^2 - 8x = -12$

Completing square, $x^2 - 8x + 16 = 4$

Extracting root, $x - 4 = \pm 2$

$\therefore \quad x = 6$ or 2, *Ans.*

7. Given $5x^2 - 40x = 45$

Dividing by 5, $x^2 - 8x = 9$

Comp. sq., $x^2 - 8x + 16 = 9 + 16 = 25$

Extracting root, $x - 4 = \pm 5$

$\therefore \quad x = 4 \pm 5$

$= 9$ or -1, *Ans.*

8. Given $x^2 - 6ax = d$

Comp. sq., $x^2 - 6ax + 9a^2 = d + 9a^2$

Extracting root, $x - 3a = \pm\sqrt{d+9a^2}$

$\therefore \quad x = 3a \pm \sqrt{d+9a^2}$, *Ans.*

10. Given $2x^2 - 22x = 120$

Dividing by 2, $x^2 - 11x = 60$

Comp. sq., $x^2 - 11x + \frac{121}{4} = 60 + \frac{121}{4} = \frac{361}{4}$

Extracting root, $x - \frac{11}{2} = \pm\frac{19}{2}$

$\therefore \quad x = \frac{11}{2} \pm \frac{19}{2}$

$= 15$ or -4, *Ans.*

11. Given $x^2 - 140 = 13x$

Transposing, $x^2 - 13x = 140$

Comp. sq., $x^2 - 13x + \frac{169}{4} = 140 + \frac{169}{4} = \frac{729}{4}$

Extracting root, $x - \frac{13}{2} = \pm\frac{27}{2}$

$\therefore \quad x = \frac{13}{2} \pm \frac{27}{2}$

$= 20$ or -7, *Ans.*

Second Method, Page 179.

2. Given $3x^2 - 9x - 3 = 207$

Transposing, etc., $x^2 - 3x = 70$

Writing value by rule, $x = \frac{3}{2} \pm \sqrt{70 + \frac{9}{4}}$

Reducing, $x = \frac{3}{2} \pm \sqrt{\frac{289}{4}} = \frac{3}{2} \pm \frac{17}{2}$

$= 10$ or -7, *Ans.*

3. Given $4x^2 + 12x + 5 = 45$

Dividing by 4, $x^2 + 3x = 10$

By the rule, $x = -\frac{3}{2} \pm \sqrt{10 + \frac{9}{4}}$

Reducing, $x = -\frac{3}{2} \pm \sqrt{\frac{49}{4}}$

$\therefore \quad x = -\frac{3}{2} \pm \frac{7}{2}$

$= 2$ or -5, *Ans.*

4. Given $3x^2 - 14x + 15 = 0$
Dividing by 3, $x^2 - \frac{14}{3}x = -5$
By the rule, $x = \frac{7}{3} \pm \sqrt{-5 + \frac{49}{9}}$
Reducing, $x = \frac{7}{3} \pm \sqrt{\frac{4}{9}}$
$\therefore \quad x = \frac{7}{3} \pm \frac{2}{3} = 3$ or $1\frac{2}{3}$.

5. Given $4x^2 - 9x = 28$
Dividing by 4, $x^2 - \frac{9}{4}x = 7$
By the rule, $x = \frac{9}{8} \pm \sqrt{7 + \frac{81}{64}}$
$= \frac{9}{8} \pm \sqrt{\frac{529}{64}}$
$\therefore \quad x = \frac{9}{8} \pm \frac{23}{8} = 4$ or $-1\frac{3}{4}$.

6. Given $\frac{x+2}{2x} + \frac{2x}{x+2} = 2$
Clearing of fractions, $x^2 + 4x + 4 + 4x^2 = 4x^2 + 8x$
Reducing, $x^2 - 4x + 4 = 0$
Extracting root, $x - 2 = 0$
$\therefore \quad x = 2$, *Ans.*

7. Given $x^2 + \frac{ax}{b} - ab = d$

Transposing, etc., $x = -\frac{a}{2b} \pm \sqrt{ab + d + \frac{a^2}{4b^2}}$, *Ans.*

8. Given $x^2 + 4ax = b$
By the rule, $x = -2a \pm \sqrt{b + 4a^2}$, *Ans.*

9. Given $3x^2 - 74 = 6x + 31$
Transposing, $3x^2 - 6x = 105$
Dividing by 3, $x^2 - 2x = 35$
$\therefore \quad x = 1 \pm \sqrt{35 + 1} = 1 \pm \sqrt{36}$
$= 1 \pm 6 = 7$ or -5, *Ans.*

10. Given $x^2 + 13 = 6x$
Transposing, $x^2 - 6x = -13$
By the rule, $x = 3 \pm \sqrt{-13 + 9}$
Reducing, $x = 3 \pm \sqrt{-4}$
$\therefore \quad x = 3 \pm \sqrt{4(-1)}$
$= 3 \pm 2\sqrt{-1}$, *Ans.*

11. Given $(x-2)(x-1) = 20$
Or $x^2 - 3x + 2 = 20$
Transposing, $x^2 - 3x = 18$
By the rule, $x = \frac{3}{2} \pm \sqrt{18 + \frac{9}{4}} = \frac{3}{2} \pm \sqrt{\frac{81}{4}}$
$\therefore \quad x = \frac{3}{2} \pm \frac{9}{2} = 6 \text{ or } -3$, *Ans.*

12. Given $\frac{x+1}{x} + \frac{x}{x+1} = 1\frac{13}{6}$
Clear. of frac., $6x^2 + 12x + 6 + 6x^2 = 13x^2 + 13x$
Transposing, etc., $x^2 + x = 6$
By the rule, $x = -\frac{1}{2} \pm \sqrt{6 + \frac{1}{4}}$
Reducing, $x = -\frac{1}{2} \pm \frac{5}{2}$
$= 2 \text{ or } -3$, *Ans.*

13. Given $x^2 - \frac{bx}{c} + ch = bd$
Transposing, etc., $x = \frac{b}{2c} \pm \sqrt{bd - ch + \frac{b^2}{4c^2}}$, *Ans.*

Third Method, Page 181.

3. Given $3x^2 + 4x = 39$
Completing sq., $9x^2 + 12x + 4 = 121$
Extracting root, $3x + 2 = \pm 11$
$\therefore \quad x = 3 \text{ or } -4\frac{1}{3}$, *Ans.*

4. Given $x^2 - 30 = -x$
Transposing, $x^2 + x = 30$
Completing sq., $4x^2 + 4x + 1 = 120 + 1 = 121$
Extracting root, $2x + 1 = \pm 11$
Transposing, $2x = -1 \pm 11$
$\therefore \quad x = 5 \text{ or } -6$, *Ans.*

5. Given $5x + 3x^2 = 2$
Or $3x^2 + 5x = 2$
Comp. sq., $36x^2 + 60x + 25 = 24 + 25 = 49$
Extracting root, $6x + 5 = \pm 7$
Transposing, $6x = -5 \pm 7$
$\therefore \quad x = \frac{1}{3} \text{ or } -2$, *Ans.*

6. Given $4x^2 - 7x - 2 = 0$
Or $4x^2 - 7x = 2$
Comp. sq., $64x^2 - 112x + 49 = 32 + 49 = 81$
Extracting root, $8x - 7 = \pm 9$
Whence, $8x = 7 \pm 9$
$\therefore \quad x = 2$ or $-\frac{1}{4}$, *Ans.*

7. Given $5x^2 + 2x = 88$
Completing sq., $25x^2 + 10x + 1 = 440 + 1 = 441$
Extracting root, $5x + 1 = \pm 21$
Whence, $5x = -1 \pm 21$
$\therefore \quad x = 4$ or $-4\frac{2}{5}$, *Ans.*

8. Given $2x^2 - 6x = 8$
Completing sq., $4x^2 - 12x + 9 = 16 + 9 = 25$
Extracting root, $2x - 3 = \pm 5$
Whence, $2x = 3 \pm 5$
$\therefore \quad x = 4$ or -1, *Ans.*

9. Given $3x^2 + 5x = 42$
Comp. sq., $36x^2 + 60x + 25 = 504 + 25 = 529$
Extracting root, $6x + 5 = \pm 23$
Whence, $6x = -5 \pm 23$
$\therefore \quad x = 3$ or $-4\frac{2}{3}$, *Ans.*

10. Given $x^2 - 15x = -54$
Comp. sq., $4x^2 - 60x + 225 = -216 + 225 = 9$
Extracting root, $2x - 15 = \pm 3$
Whence, $2x = 15 \pm 3$
$\therefore \quad x = 9$ or 6, *Ans.*

11. Given $9x^2 - 7x = 116$
Comp. sq., $324x^2 - 252x + 49 = 4176 + 49 = 4225$
Extracting root, $18x - 7 = \pm 65$
Whence, $18x = 7 \pm 65$
$\therefore \quad x = 4$ or $-3\frac{2}{9}$, *Ans.*

EXAMPLES.

Page 182.

1. Given $x^2 - 4x = -3$
Completing square, etc., $x = 2 \pm \sqrt{-3+4}$
Reducing, $x = 2 \pm 1$
$\therefore \quad x = 3$ or 1, *Ans.*

2. Given $x^2 - 5x = -4$
Completing square, $x = \frac{5}{2} \pm \sqrt{-4+\frac{25}{4}}$
Extracting root, $x = \frac{5}{2} \pm \frac{3}{2}$
$\therefore \quad x = 4$ or 1, *Ans.*

3. Given $2x^2 - 7x = -3$
Comp. sq., $16x^2 - 56x + 49 = -24 + 49 = 25$
Extracting root, $4x - 7 = \pm 5$
Whence, $4x = 7 \pm 5$
$\therefore \quad x = 3$ or $\frac{1}{2}$, *Ans.*

4. Given $x^2 + 10x = 24$
Completing square, etc., $x = -5 \pm \sqrt{24+25}$
Reducing, $x = -5 \pm 7$
$\therefore \quad x = 2$ or -12, *Ans.*

5. Given $6x^2 - 13x + 6 = 0$
Or, $6x^2 - 13x = -6$
Dividing by 6, $x^2 - \frac{13}{6}x = -1$
By 2d method, $x = \frac{13}{12} \pm \sqrt{-1+\frac{169}{144}}$
Reducing, $x = \frac{13}{12} \pm \sqrt{\frac{25}{144}}$
Or, $x = \frac{13}{12} \pm \frac{5}{12}$
$\therefore \quad x = 1\frac{1}{2}$ or $\frac{2}{3}$, *Ans.*

6. Given $14x - x^2 = 33$
Changing signs, $x^2 - 14x = -33$
By 2d method, $x = 7 \pm \sqrt{-33+49}$
$\therefore \quad x = 7 \pm 4$
$= 11$ or 3, *Ans.*

7. Given $x^2 - 3 = \frac{x-3}{6}$

Multiplying by 6, $6x^2 - 18 = x - 3$

Transposing, $6x^2 - x = 15$

By 3d method, $144x^2 - 24x + 1 = 360 + 1 = 361$

Extracting root, $12x - 1 = \pm 19$

Whence, $12x = 1 \pm 19$

$\therefore \quad x = 1\frac{2}{3}$ or $-1\frac{1}{2}$, *Ans.*

8. Given $\frac{5x^2}{7} + \frac{7x}{5} = -\frac{73}{140}$

Mult. by 140, $100x^2 + 196x = -73$

By 3d method,

$10000x^2 + 19600x + (98)^2 = -7300 + 9604$

$= 2304$

Extracting root, $100x + 98 = \pm 48$

Whence $100x = -98 \pm 48$

$\therefore \quad x = -\frac{1}{2}$ or $-1\frac{23}{50}$.

9. Given $\frac{16}{x} - \frac{100 - 9x}{4x^2} = 3$

Clearing of frac., $64x - 100 + 9x = 12x^2$

Transposing, $12x^2 - 73x = -100$

By 3d method,

$576x^2 - 3504x + (73)^2 = -4800 + 5329$

$= 529$

Extracting root, $24x - 73 = \pm 23$

Whence, $24x = 73 \pm 23$

$\therefore \quad x = 4$ or $2\frac{1}{12}$, *Ans.*

10. Given $\frac{a}{x} + \frac{x}{a} = \frac{2}{a}$

Mult. by ax, $a^2 + x^2 = 2x$

Transposing, $x^2 - 2x = -a^2$

By 2d method, $x = 1 \pm \sqrt{-a^2 + 1}$, *Ans.*

11. Given $x^2 + 2mx = b^2$

By 2d method, $x = -m \pm \sqrt{b^2 + m^2}$, *Ans.*

12. Given $x^2 + \frac{x}{8} = 1\frac{1}{8}$

By 2d method, $x = -\frac{1}{16} \pm \sqrt{\frac{9}{8} + \frac{1}{256}}$

$= -\frac{1}{16} \pm \sqrt{\frac{289}{256}}$

Reducing, $x = -\frac{1}{16} \pm \frac{17}{16}$

$\therefore\ x = 1$ or $-1\frac{1}{8}$, *Ans.*

13. Given $\frac{x^3 - 10x^2 + 1}{x^2 - 6x + 9} = x - 3$

Mult. by denom., $x^3 - 10x^2 + 1 = x^3 - 9x^2 + 27x - 27$

Transposing, $x^2 + 27x = 28$

By 2d method, $x = -\frac{27}{2} \pm \sqrt{28 + \frac{729}{4}}$

$= -\frac{27}{2} \pm \sqrt{\frac{841}{4}}$

Reducing, $x = -\frac{27}{2} \pm \frac{29}{2}$

$\therefore\ x = 1$ or -28, *Ans.*

14. Given $\frac{4x}{14 - x} - \frac{x - 1}{3x} = \frac{9x + 7}{x}$

Clearing of fractions,

$12x^2 - 14x + x^2 + 14 - x = 378x - 27x^2 + 294 - 21x$

Transposing, $40x^2 - 372x = 280$

By 3d method,

$400x^2 - 3720x + (93)^2 = 2800 + 8649 = 11449$

Extracting root, $20x - 93 = \pm 107$

Whence, $20x = 93 \pm 107$

$\therefore\ x = 10$ or $-\frac{7}{10}$, *Ans.*

15. Given $2\sqrt{x^2 - 4x} - 1 = -4x$

Transposing, $2\sqrt{x^2 - 4x} = 1 - 4x$

Squaring, $4x^2 - 16x = 1 - 8x + 16x^2$

Transposing, $12x^2 + 8x = -1$

Dividing by 12, $x^2 + \frac{8}{12}x = -\frac{1}{12}$

By 2d method, $x = -\frac{1}{3} \pm \sqrt{-\frac{1}{12} + \frac{1}{9}}$

$= -\frac{1}{3} \pm \sqrt{\frac{1}{36}}$

$\therefore\ x = -\frac{1}{3} \pm \frac{1}{6}$

$= -\frac{1}{6}$ or $-\frac{1}{2}$, *Ans.*

16. Given $\sqrt{x+5}+6=x+5$

Transposing, $\sqrt{x+5}=x-1$

Squaring, $x+5=x^2-2x+1$

Transposing, $x^2-3x=4$

By 2d method, $x=\frac{3}{2}\pm\sqrt{4+\frac{9}{4}}=\frac{3}{2}\pm\frac{5}{2}$

$=4$ or -1, *Ans.*

SECOND SOLUTION.

$\sqrt{x+5}+6=x+5$

Transposing, $x+5-\sqrt{x+5}=6$

By 2d method, $\sqrt{x+5}=\frac{1}{2}\pm\sqrt{6+\frac{1}{4}}$

Reducing, $\sqrt{x+5}=\frac{1}{2}\pm\frac{5}{2}$

Uniting, $\sqrt{x+5}=3$ or -2

Squaring, $x+5=9$ or 4

$\therefore\quad x=4$ or -1, *Ans.*

NOTE.—In the above solution, $x+5$ is regarded as a simple unknown quantity, whose value is to be found. For beginners, it might be more intelligible to substitute y for it. Then the equation would be $y-\sqrt{y}=6$, which can be readily solved as above.

17. Given $3x^2-7x-20=0$

Or, $3x^2-7x=20$

By 3d method, $36x^2-84x+49=240+49=289$

Extracting root, $6x-7=\pm 17$

Whence, $6x=7\pm 17$

$\therefore\quad x=4$ or $-1\frac{2}{3}$, *Ans.*

18. Given $7x^2-160=3x$

Transposing, $7x^2-3x=160$

By 3d method, $196x^2-84x+9=4480+9=4489$

Extracting root, $14x-3=\pm 67$

Whence, $14x=3\pm 67$

$\therefore\quad x=5$ or $-4\frac{4}{7}$, *Ans.*

19. Given $2x^2 - 2x = 1\frac{1}{2}$
Multiplying by 2, $4x^2 - 4x = 3$
By 3d method, $4x^2 - 4x + 1 = 4$
Extracting root, $2x - 1 = \pm 2$
Whence, $2x = 1 \pm 2$
$\therefore \quad x = 1\frac{1}{2}$ or $-\frac{1}{2}$, *Ans.*

20. Given $(x - 2)(x - 1) = 6$
Reducing, $x^2 - 3x = 4$
By 2d method, $x = \frac{3}{2} \pm \sqrt{4 + \frac{9}{4}}$
Or, $x = \frac{3}{2} \pm \frac{5}{2}$
$\therefore \quad x = 4$ or -1, *Ans.*

21. Given $4(x^2 - 1) = 4x - 1$
Or, $4x^2 - 4x = 3$
(See Ex. 19.) $\therefore \quad x = 1\frac{1}{2}$ or $-\frac{1}{2}$, *Ans.*

22. Given $(2x - 3)^2 = 8x$
Reducing, $4x^2 - 20x = -9$
By 3d method, $4x^2 - 20x + 25 = -9 + 25 = 16$
Extracting root, $2x - 5 = \pm 4$
Whence, $2x = 5 \pm 4$
$\therefore \quad x = 4\frac{1}{2}$ or $\frac{1}{2}$, *Ans.*

23. Given $3x - 2 = \frac{14}{x - 1}$
Mult. by $x - 1$, etc., $3x^2 - 5x = 12$
By 3d meth., $36x^2 - 60x + 25 = 144 + 25 = 169$
Extracting root, $6x - 5 = \pm 13$
Whence, $6x = 5 \pm 13$
$\therefore \quad x = 3$ or $-1\frac{1}{3}$, *Ans.*

24. Given $4x - \frac{14 - x}{x + 1} = 14$
Mult. by $x + 1$, etc., $4x^2 - 9x = 28$
By 3d meth., $64x^2 - 144x + 81 = 448 + 81 = 529$
Extracting root, $8x - 9 = \pm 23$
Whence, $8x = 9 \pm 23$
$\therefore \quad x = 4$ or $-1\frac{1}{4}$, *Ans.*

25. Given $x^2 + \frac{3}{25} = \frac{4x}{5}$

Transposing, $x^2 - \frac{4x}{5} = -\frac{3}{25}$

By 2d method, $x = \frac{2}{5} \pm \sqrt{-\frac{3}{25} + \frac{4}{25}}$

Reducing, $x = \frac{2}{5} \pm \frac{1}{5}$

$\therefore \quad x = \frac{3}{5}$ or $\frac{1}{5}$, *Ans.*

26. Given $x^2 + \frac{x}{2} = \frac{1}{2}$

By 2d method, $x = -\frac{1}{4} \pm \sqrt{\frac{1}{2} + \frac{1}{16}} = -\frac{1}{4} \pm \sqrt{\frac{9}{16}}$

Reducing, $x = -\frac{1}{4} \pm \frac{3}{4}$

$\therefore \quad x = \frac{1}{2}$ or -1, *Ans.*

27. Given $x^2 - 2nx = m^2 - n^2$

By 2d method, $x = n \pm \sqrt{m^2 - n^2 + n^2}$

$\therefore \quad x = n \pm m$, *Ans.*

28. Given $\frac{9(b-a)}{x} = \frac{x - 3a}{b}$

Clearing of fractions, $9b^2 - 9ab = x^2 - 3ax$

Transposing, $x^2 - 3ax = 9b^2 - 9ab$

By 2d method, $x = \frac{3a}{2} \pm \sqrt{9b^2 - 9ab + \frac{9a^2}{4}}$

Reducing, $x = \frac{3a}{2} \pm \sqrt{\frac{36b^2 - 36ab + 9a^2}{4}}$

Or, $x = \frac{3a}{2} \pm \frac{6b - 3a}{2}$

$\therefore \quad x = 3b$ or $3a - 3b$, *Ans.*

EQUATIONS OF THE QUADRATIC FORM.

Page 183.

4. Given $x^4 + 8 = 6x^2$
Transposing, $x^4 - 6x^2 = -8$
By 2d method, $x^2 = 3 \pm \sqrt{-8+9}$
Reducing, $x^2 = 3 \pm 1$
Uniting, $x^2 = 4$ or 2
Extracting root, $x = \pm 2$ or $\pm \sqrt{2}$, *Ans.*

5. Given $x^4 - 2x^2 = 3$
By 2d method, $x^2 = 1 \pm \sqrt{3+1}$
Reducing, $x^2 = 1 \pm 2$
Uniting, $x^2 = 3$ or -1
Extracting root, $x = \pm \sqrt{3}$ or $\pm \sqrt{-1}$, *Ans.*

6. Given $x^6 - 7x^3 = 0$
Transposing, $x^6 = 7x^3$
Dividing by x^3, $x^3 = 7$
Extracting cube root, $x = \sqrt[3]{7} = 1.91+$, *Ans.*

7. Given $\frac{x^2}{2} + \frac{x}{4} = \frac{3}{32}$
Multiplied by 2, $x^2 + \frac{x}{2} = \frac{3}{16}$
By 2d method, $x = -\frac{1}{4} \pm \sqrt{\frac{3}{16} + \frac{1}{16}}$
Reducing, $x = -\frac{1}{4} \pm \frac{2}{4}$
$\therefore \quad x = \frac{1}{4}$ or $-\frac{3}{4}$, *Ans.*

8. Given $\sqrt[3]{x^2} + \frac{3}{2}\sqrt[3]{x} = 1$
By 2d method, $\sqrt[3]{x} = -\frac{3}{4} \pm \sqrt{1 + \frac{9}{16}}$
Reducing, $\sqrt[3]{x} = -\frac{3}{4} \pm \frac{5}{4}$
Uniting, $\sqrt[3]{x} = \frac{1}{2}$ or -2
Involving, $x = \frac{1}{8}$ or -8, *Ans.*

9. Given $4x + 4\sqrt{x+2} = 7$

Transposing, $4\sqrt{x+2} = 7 - 4x$

Squaring, $16x + 32 = 49 - 56x + 16x^2$

Transposing, $16x^2 - 72x = -17$

By 3d me., $256x^2 - 1152x + (36)^2 = -272 + 1296$

$= 1024$

Extracting root, $16x - 36 = \pm 32$

Whence, $16x = 36 \pm 32$

$\therefore \quad x = 4\frac{1}{4}$ or $\frac{1}{4}$, *Ans.*

10. Given $\dfrac{\sqrt{4x+20}}{4+\sqrt{x}} = \dfrac{4-\sqrt{x}}{\sqrt{x}}$

Clearing of fractions and squaring,

$4x^2 + 20x = 256 - 32x + x^2$

Transposing, $3x^2 + 52x = 256$

By 3d meth., $9x^2 + 156x + (26)^2 = 768 + 676 = 1444$

Extracting root, $3x + 26 = \pm 38$

Whence, $3x = -26 \pm 38$

$\therefore \quad x = 4$ or $-21\frac{1}{3}$, *Ans.*

PROBLEMS.

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1. Let $x =$ One,

Then $12 - x =$ The other.

And $12x - x^2 = 32$

Or $x^2 - 12x = -32$

Comp. sq., $x^2 - 12x + 36 = -32 + 36$ (Art. 333.)

Extracting root, $x - 6 = \pm 2$

Whence, $x = 8$ or 4; $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ *Ans.*

$12 - x = 4$ or 8, $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$

2. Let $x =$ Cost,

Then $\dfrac{x}{100} =$ Rate per cent,

And $\dfrac{x^2}{100} =$ Percentage lost.

By conditions, $x - \frac{x^2}{100} = 24$

Clearing, $100x - x^2 = 2400$

Changing signs, $x^2 - 100x = -2400$

By 2d method, $x = 50 \pm \sqrt{-2400 + 2500}$

$\therefore \quad x = 50 \pm 10$

$= \$60$ or $\$40$, *Ans.*

3. Let $x =$ one number,

And $y =$ the other.

Then $x + y = 10$ (1)

And $xy = 24$ (2)

Squaring (1), $x^2 + 2xy + y^2 = 100$ (3)

Mult. (2) by 4, $4xy = 96$ (4)

Sub. (4) from (3), $x^2 - 2xy + y^2 = 4$ (5)

Extracting root, $x - y = \pm 2$ (6)

Bringing down (1) $x + y = 10$

Adding (6) and (1), etc., $x = 6$ or 4;

Subtracting (6) from (1), etc., $y = 4$ or 6, } *Ans.*

NOTE.—Let the pupil compare this solution with that of Problem 1, in which one unknown quantity is used.

4. Let $x =$ no. of sheep,

Then $\frac{80}{x} =$ price of each.

And $\frac{80}{x} = \frac{80}{x+4} + 1$

Clearing of fractions, $80x + 320 = 80x + x^2 + 4x$

Rejecting $80x$ from each, $x^2 + 4x = 320$

Completing square, $x^2 + 4x + 4 = 324$

Extracting root, $x + 2 = \pm 18$

Whence, $x = -2 \pm 18$

$\therefore \quad x = 16$ sheep }

And $\frac{80}{x} = \$5$ each, } *Ans.*

The problem will not admit of a negative result. Therefore the *Ans.* 16 sheep, at $\$5$ a head.

5. Let $x =$ the number.
Then $2x^2 = 65 - 3x$
Transposing, $2x^2 + 3x = 65$
Dividing by 2, $x^2 + \frac{3}{2}x = \frac{65}{2}$
Completing square, etc., $x = -\frac{3}{4} \pm \sqrt{\frac{65}{2} + \frac{9}{16}}$
$\therefore \quad x = -\frac{3}{4} \pm \frac{23}{4}$
$= 5$ or $-6\frac{1}{2}$, *Ans.*

6. Let $x =$ No. of scholars,
Then $\frac{144}{x} =$ " " oranges each received.
And $\frac{144}{x} = \frac{144}{x+2} + 1$
Clearing of frac., $144x + 288 = 144x + x^2 + 2x$
Or $x^2 + 2x = 288$
By 2d method, $x = -1 \pm \sqrt{288+1}$
$\therefore \quad x = -1 \pm 17$
$= 16$ or -18.

NOTE.—The second value of x, being negative, cannot be applied to children. It may be observed that negative values do not apply to concrete numbers. Hence the *Ans.* 16 scholars.

7. Let $x =$ share of one,
Then $50 - x =$ " " the other.
And $50x - x^2 = 600$
Changing signs, $x^2 - 50x = -600$
By 2d method, $x = 25 \pm \sqrt{-600 + 625}$
$\therefore \quad x =$ \$30, one; } *Ans.*
And $50 - x =$ \$20, other, }

8. Let $x =$ one,
Then $100 - x =$ the other.
And $100x - x^2 = 2400$
Changing signs, $x^2 - 100x = -2400$
By 2d method, $x = 50 \pm \sqrt{-2400 + 2500}$
$\therefore \quad x = 60$; } *Ans.*
And $100 - x = 40$, }

9. Half the perimeter ($\frac{128}{2} = 64$), or 64 rods, is equal to the *sum* of the length and breadth.

Let $x =$ the length,

Then $64 - x =$ " breadth.

And $64x - x^2 = 1008$ sq. rds., area.

Changing signs, $x^2 - 64x = -1008$

By 2d method, $x = 32 \pm \sqrt{-1008 + 1024}$

$\therefore \quad x = 36$ rds., l'gth;
$64 - x = 28$ " br'dth, } *Ans.*

10. Let $x =$ number in file,

Then $x + 60 =$ " " rank.

And $x^2 + 60x = 1600$ men.

By 2d method, $x = -30 \pm \sqrt{1600 + 900}$

Reducing, $x = -30 \pm 50$

$\therefore \quad x = 20$ men in file;
$x + 60 = 80$ " " rank, } *Ans.*

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11. Let $x =$ number of lambs.

Then $\frac{50}{x} =$ cost of each.

And $5\frac{1}{2}x$, or $\frac{11x}{2} =$ amt. received for them.

By conditions, $\frac{11x}{2} - 50 = \frac{50}{x}$

Clearing of frac., $11x^2 - 100x = 100$

By 3d method,

$484x^2 - 4400x + 10000 = 14400$. (Art. 336.)

Extracting root, $22x - 100 = \pm 120$

Transposing and dividing, $x = 10$ lambs, *Ans.*

12. Let x = one number,
Then $4 - x$ = the other.
And $\frac{1}{x} + \frac{1}{4-x} = 1$
Clearing of frac., etc., $x^2 - 4x = -4$
By second method, $x = 2$;
And $4 - x = 2$, } *Ans.*

SECOND SOLUTION.

Denote the numbers by x and y.

Then $x + y = 4$ (1)
And $\frac{1}{x} + \frac{1}{y} = 1$ (2)
Clearing (2), $x + y = xy$ (3)
Subtracting (3) from (1), $0 = 4 - xy$
Or, $xy = 4$ (4)

From (1) and (4) find the value of $x - y$, thus:

$x + y = 4$ (5)
Squaring (1), $x^2 + 2xy + y^2 = 16$ (6)
Mult. (4) by 4, $4xy = 16$ (7)
Subt. (7) from (6), $x^2 - 2xy + y^2 = 0$
Extracting root, $x - y = 0$
Or, $x = y$ (8)
Substituting in (1), $2x = 4$
$x = 2$
From (8), $y = 2$. *Ans.* 2 and 2.

13. Denote the numbers by x and y.

Then $x + y = 5$ (1)
And $x^3 + y^3 = 65$ (2)
Dividing (2) by (1), $x^2 - xy + y^2 = 13$. (Ax. 5.) (3)
Squaring (1), $x^2 + 2xy + y^2 = 25$ (4)
Subt. (3) from (4), $3xy = 12$ (5)

Or, $xy = 4$

Whence, $y = \frac{4}{x}$ (6)

Substituting in (1), $x + \frac{4}{x} = 5$

Clearing, etc., $x^2 - 5x = -4$

By 2d method, $x = \frac{5}{2} \pm \sqrt{-4 + \frac{25}{4}}$

$\therefore \quad x = \frac{5}{2} \pm \frac{3}{2} = 4$ or 1

Substituting in (6), $y = 1$ or 4.

Ans. 4 and 1.

This problem may be solved with one unknown quantity, thus,

Let $x =$ one number.

Then $5 - x =$ the other.

And $125 - 75x + 15x^2 - x^3 + x^3 = 65$

Transposing, etc., $15x^2 - 75x = -60$

Or, $x^2 - 5x = -4$

Completing sq., etc., $x = \frac{5}{2} \pm \sqrt{-4 + \frac{25}{4}}$

$\therefore \quad x = 4$ or 1

And $5 - x = 1$ or 4

Ans. 4 and 1.

14. Let $x =$ No. of yards in the width,

Then $x + 1 =$ " " " length.

1 acre = 4840 square yards.

3 acres = 14520 " "

Length × width $= x^2 + x = 14520$

By 2d method, $x = -\frac{1}{2} \pm \sqrt{14520 + \frac{1}{4}}$

Reducing, $x = -\frac{1}{2} \pm \sqrt{\frac{58081}{4}}$

Or, $x = -\frac{1}{2} \pm \frac{241}{2}$

$\therefore \quad x = 120$ yards, width;
$x + 1 = 121$ " length, } *Ans.*

15. Let x = B's rate,

Then $x + 1$ = A's "

$\frac{300}{x}$ = time it takes B to go.

$\frac{300}{x+1}$ = " " A "

And $\frac{300}{x} = \frac{300}{x+1} + 10$

Dividing by 10, clearing, etc.,

$x^2 + x = 30$

By 2d meth., $x = -\frac{1}{2} \pm \sqrt{30 + \frac{1}{4}}$

Reducing, $x = -\frac{1}{2} \pm \frac{11}{2}$

$\therefore \quad x = 5$ miles, B's rate;

And $x + 1 = 6$ " A's " } *Ans.*

16. Let x = No. that B relieves,

Then $x + 40$ = " " A "

$\frac{1200}{x}$ = sum B gives to each,

$\frac{1200}{x+40}$ = " A " "

By conditions, $\frac{1200}{x} = \frac{1200}{x+40} + 5$

Dividing by 5, $\frac{240}{x} = \frac{240}{x+40} + 1$

Clear. of frac., $240x + 9600 = 240x + x^2 + 40x$

Transposing, $x^2 + 40x = 9600$

By 2d method, $x = -20 \pm \sqrt{9600 + 400}$

Reducing, $x = -20 \pm 100$

$\therefore \quad x = 80$

Whence, $x + 40 = 120$

80 relieved by B;

And 120 " " A, } *Ans.*

17. Let x = one part,
Then $48 - x$ = the other.
And $x^2 - 48x = -252$
Completing square, etc., *Ans.* 42 and 6.

BY TWO UNKNOWN QUANTITIES.

Denote the parts by x and y.

Then	$x + y = 48$	(1)
	$xy = 252$	(2)
Squaring (1),	$(x+y)^2 = 2304$	(3)
Multiplying (2) by 4,	$4xy = 1008$	(4)
Subtracting,	$(x-y)^2 = 1296$	(5)
Extracting root,	$x - y = \pm 36$	(6)
	$x + y = 48$	(1)
Adding (1) to (6), etc.,	$x = 42$ or 6;	} *Ans.*
Subt. (6) from (1), etc.,	$y = 6$ or 42.	}

18. Let x = number A bought,
And y = " B "
Then $x + y = 10$ (1)
$\frac{12}{x}$ = cents A paid apiece. (2)
$\frac{12}{y}$ = " B " " (3)
By conditions, $\frac{12}{x} = \frac{12}{y} + 1$ (4)
Clearing of frac., $12y = 12x + xy$ (5)
Taking value of x from (1),
$12y = 12(10-y) + y(10-y)$
Reducing, $12y = 120 - 12y + 10y - y^2$
Transposing, $y^2 + 14y = 120$
By 2d method, $y = -7 \pm \sqrt{169}$
Reducing, $y = -7 \pm 13$
$\therefore$ $y = 6$ lemons, B; } *Ans.*
From (1), $x = 4$ " A, }

BY ONE UNKNOWN QUANTITY.

Let $x =$ No. A bought,

Then $10 - x =$ " B "

By conditions, $\frac{12}{x} = \frac{12}{10 - x} + 1$

Clearing of frac., etc., $x^2 - 34x = -120$

By 2d method, $x = 17 \pm \sqrt{-120 + 289}$

$\therefore \quad x = 17 \pm 13 = 4$, A; And $10 - x = 6$, B, } *Ans.*

19. Let $x =$ the breadth.

Then $24 - x =$ the length.

By conditions, $24x - x^2 = 35\,(24 - 2x)$

Reducing, $x^2 - 94x = -840$, etc.

Ans. 14 ft. length; 10 ft. breadth.

BY TWO UNKNOWN QUANTITIES.

Let $x =$ the length,

And $y =$ the breadth.

Then $x + y = 24$ ft., or half the perimeter. (1)

$xy = 35\,(x - y)$, the area. (2)

Mult. (1) by 35, $35x + 35y = 840$ (3)

Transposing (2), $35x - 35y = xy$ (4)

Subt. (4) from (3), $70y = 840 - xy$

Taking val. of x from (1), $70y = 840 - (24 - y)\,y$

Reducing, $70y = 840 - 24y + y^2$

Transposing, etc., $y^2 - 94y = -840$

By 2d method, $y = 47 \pm \sqrt{-840 + 2209}$

$y = 47 \pm 37$

$\therefore \quad y = 10$, breadth; And $x = 14$, length. } *Ans.*

20. Let x = No. of rows.
Then $x + 3$ = " " trees in each row.
And $x^2 + 3x = 180$
By 2d method, $x = -\frac{3}{2} \pm \sqrt{180 + \frac{9}{4}}$
Reducing, $x = -\frac{3}{2} \pm \frac{27}{2}$
$\therefore \quad x = 12$ rows;
And $x + 3 = 15$ trees in each, } *Ans.*

21. Let x = digit in tens' place,
Then $7 - x$ = " " units' "
And $x^2 + (7 - x)^2 = 29$
Reducing, $x^2 - 7x = -10$
Completing square, etc., *Ans.* 52.

BY TWO UNKNOWN QUANTITIES.

Let $10x + y$ = number.
Then $x + y = 7$ (1)
And $x^2 + y^2 = 29$ (2)
Squaring (1), $x^2 + 2xy + y^2 = 49$ (3)
Subt. (2) from (3), $2xy = 20$ (4)
Combining (2) and (4), $x - y = 3$ (5)
" (5) " (1), $x = 5$
$y = 2$
(See solution of Ex. 17.) $10x + y = 52$, *Ans.*

22. Let x = No. in the party,
Then $x + 30$ = contribution required of each.
And $x^2 + 30x = 1000$
By 2d method, $x = -15 \pm \sqrt{1000 + 225}$
Reducing, $x = -15 \pm 35$
$\therefore \quad x = 20$ persons, *Ans.*

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23. Let x = the less,

Then $\frac{120}{x}$ = " greater.

And $(x+2)\left(\frac{120}{x}-3\right) = 120$

Reducing, $x^2 + 2x = 80$, etc.

Ans. 8 and 15.

BY TWO UNKNOWN QUANTITIES.

Let x = the greater, and y = the less.

Then $xy = 120$ (1)

And $(y+2)(x-3) = 120$ (2)

From (2), $xy + 2x - 3y - 6 = 120$ (3)

From (1), $xy = 120$, and $x = \frac{120}{y}$ (4)

From (3), $120 + \frac{240}{y} - 3y = 126$ (5)

Clearing, etc., $3y^2 + 6y = 240$

Dividing by 3, $y^2 + 2y = 80$

By 2d method, $y = -1 \pm \sqrt{81}$

Reducing, $y = -1 \pm 9$

$\therefore$ $y = 8$ or -10, less;

From (1), $x = 15$ or -12, greater, } *Ans.*

24. Let x = one,

Then $36 - x$ = other.

By the conditions, $36x - x^2 = 80(36 - 2x)$

Reducing, $x^2 - 196x = -2880$

Completing sq., etc., $x = 98 \pm \sqrt{-2880 + 9604}$

Or $x = 98 \pm \sqrt{6724}$

And $x = 98 \pm 82$

$\therefore$ $x = 16$;

And $36 - x = 20$, } *Ans.*

25. Denote the numbers by x and y.

Then $x + y = 75$ (1)

And $xy : x^2 + y^2 :: 2 : 5$ (2)

Changing (2) to an equation,

$2x^2 + 2y^2 = 5xy$ (3)

Squaring (1), $x^2 + 2xy + y^2 = 5625$ (4)

Transposing and mult., $2x^2 + 2y^2 = 11250 - 4xy$ (5)

Equating (3) and (5), $5xy = 11250 - 4xy$ (6)

Reducing, $xy = 1250$ (7)

Combining (4) and (7), $x - y = 25$ (8)

Combining (1) and (8), $x = 50$;

Substituting in (8), $y = 25$, } *Ans.*

BY ONE UNKNOWN QUANTITY.

Let $x =$ one number,

Then $75 - x =$ the other.

And $75x - x^2 : x^2 + (75 - x)^2 :: 2 : 5$

Placing the product of the extremes equal to the product of the means and reducing,

$x^2 - 75x = -1250$, etc.

26. Denote the numbers by x and $146 - x$.

Then $\sqrt{146 - x} - \sqrt{x} = 6$

Transposing and involving,

$146 - x = 36 + 12\sqrt{x} + x$

Transposing and dividing,

$55 - x = 6\sqrt{x}$

Squaring, $3025 - 110x + x^2 = 36x$

Transposing, $x^2 - 146x = -3025$

Completing sq., etc., $x = 73 \pm \sqrt{-3025 + 5329}$

Reducing, $x = 73 \pm 48$

$\therefore \quad x = 121$ or 25;

And $146 - x = 25$ or 121, } *Ans.*

27. Let x = circumference of fore-wheel,
And y = " " hind "

Then $$\frac{3600}{x} = \frac{3600}{y} + 60 \quad (1)$$

And $$\frac{3600}{x+3} = \frac{3600}{y+3} + 40 \quad (2)$$

Dividing (1) by 60, $$\frac{60}{x} = \frac{60}{y} + 1$$

Clearing of fractions, $$60y = 60x + xy$$

Factoring and dividing, $$x = \frac{60y}{60+y} \quad (3)$$

Dividing (2) by 40, $$\frac{90}{x+3} = \frac{90}{y+3} + 1$$

Clearing of fractions,
$$90y + 270 = 90x + 270 + xy + 3x + 3y + 9$$

Transposing, etc., $$87y - 9 = 93x + xy$$

Factoring and dividing, $$x = \frac{87y-9}{93+y} \quad (4)$$

Equating (3) and (4), $$\frac{60y}{60+y} = \frac{87y-9}{93+y}$$

Clearing, $$5580y + 60y^2 = 5220y - 540 + 87y^2 - 9y$$

Transposing, etc., $$27y^2 - 369y = 540$$

By 3d method,
$$4 \times 27^2y^2 - 4 \times 27 \times 369y + \overline{369}^2 = 540 \times 108 + 136161$$

Extracting root, $$54y - 369 = \pm\sqrt{194481} = \pm 441$$

Whence, $$54y = 810$$
$$\therefore \quad y = 15 \text{ ft.};$$

From (3), $x = \frac{60 \times 15}{60+15} = \frac{60 \times 15}{75} = 12$ " } *Ans.*

BY ONE UNKNOWN QUANTITY.

Let x = revolutions of hind-wheel,
Then $60 + x$ = " " fore-wheel,
And $\frac{3600}{x}$ = circumference of hind-wheel,
And $\frac{3600}{60+x}$ = " " fore-wheel.

By the conditions, $$\frac{3600}{\frac{3600}{x}+3} = \frac{3600}{\frac{3600}{60+x}+3} - 40$$

Clearing of frac., etc., $x^2 + 2460x = 648000$

By 2d method, $x = -1230 \pm \sqrt{648000 + 1512900}$

Reducing, $x = -1230 \pm 1470$

$\therefore \quad x = 240$

$$\left.\begin{aligned} \frac{3600}{x} &= 15 \text{ ft. hind-wheel;} \\ \frac{3600}{60+x} &= 12 \text{ ft. fore-wheel,} \end{aligned}\right\} \textit{Ans.}$$

28. Let $x =$ one number,

And $x + 16 =$ the other.

By the conditions, $x^2 + 16x = 36$

Completing square, etc., $x = -8 \pm \sqrt{36 + 64}$

$\therefore \quad x = 2 \text{ or } -18;$

And $x + 16 = 18 \text{ or } -2,$ } *Ans.*

29. Denote the numbers by x and y.

Then $x + y = 1\frac{1}{3} = \frac{4}{3}$ (1)

And $\frac{1}{x} + \frac{1}{y} = 3\frac{1}{3} = \frac{16}{5}$ (2)

Multiplying (2) by xy, $x + y = \frac{16xy}{5}$ (3)

Equating (1) and (3), $\frac{16xy}{5} = \frac{4}{3}$

Multiplying by $\frac{5}{4}$, $4xy = \frac{5}{3}$ (4)

Squaring (1), $x^2 + 2xy + y^2 = \frac{16}{9}$ (5)

Subt. (4) from (5), etc., $x - y = \frac{1}{3}$ (6)

Combining (1) and (6), $x = \frac{5}{6};$

And $y = \frac{1}{2},$ } *Ans.*

BY ONE UNKNOWN QUANTITY.

Let x = one number,
Then $\frac{4}{3} - x$ = the other.

And $$\frac{1}{x} + \frac{3}{4 - 3x} = \frac{16}{5}$$

Clear. of frac., etc., $12x^2 - 16x = -5$, etc.

30. Let x = the less number,
Then $x + 15$ = " greater "

And $$\frac{x^2 + 15x}{2} = x^2$$

Clearing of frac., etc., $x + 15 = 2x^2$
Transposing, etc., $x^2 - \frac{1}{2}x = \frac{15}{2}$
Completing square, etc., $x = \frac{1}{4} \pm \sqrt{\frac{15}{2} + \frac{1}{16}}$
Reducing, $x = \frac{1}{4} \pm \sqrt{\frac{121}{16}}$
Or, $x = \frac{1}{4} \pm \frac{11}{4}$
$\therefore$ $x = 3$ less;
And $x + 15 = 18$ greater, } *Ans.*

31. Let x = her age.

Then $$\frac{x}{2} + \sqrt{x} - 12 = 0$$

Clearing of frac., etc., $x + 2\sqrt{x} = 24$
Completing square, etc., $\sqrt{x} = -1 \pm 5$
Uniting, $\sqrt{x} = 4$ or -6
Involving, $x = 16$ or 36.
Hence her age is 16, or 36 years, *Ans.*

NOTE.—The minus sign must be placed before the radical in the first equation when the second value is taken.

32. Denote the length by x and the breadth by y.
Then $2(x + y) = 96$ (1)
And $xy = 70(x - y)$ (2)
For solution, see that of Problem 19, above.

28 rods, length; } *Ans.*
20 " breadth, }

33. Let x = A's age,

And $\frac{120}{x}$ = B's "

By conditions, $(x-3)\left(\frac{120}{x}+2\right) = 120$

Multiplying, etc., $2x^2 + 114x = 120x + 360$

Transposing, etc., $x^2 - 3x = 180$

Completing sq., $x^2 - 3x + \frac{9}{4} = 180 + \frac{9}{4} = \frac{729}{4}$

Extracting root, $x - \frac{3}{2} = \pm \frac{27}{2}$

$\therefore \quad x = \frac{3}{2} \pm \frac{27}{2}$, or 15 years, A's age;

$\frac{120}{x} =$ 8 " B's " } *Ans.*

34. Let $x =$ { No. of lbs. pepper bought for 8 crowns.

Then $\frac{8}{x} =$ price of pepper per lb.

$80 \times \frac{8}{x} = \frac{640}{x} =$ sum paid for pepper.

$x - 14 =$ lbs. saffron for 26 crowns.

$\frac{26}{x-14} =$ price of saffron per lb.

$36 \times \frac{26}{x-14} = \frac{936}{x-14} =$ sum paid for saffron.

And $\frac{640}{x} + \frac{936}{x-14} = 188$, by conditions.

Clearing of fractions,

$640x - 8960 + 936x = 188x^2 - 2632x$

Transposing,

$188x^2 - 4208x = -8960$

Dividing by 4,

$47x^2 - 1052x = -2240$

By 3d, $94x - 1052 = \pm\sqrt{-2240 \times 188 + (1052)^2}$

Reducing, $94x - 1052 = \pm\sqrt{-421120 + 1106704}$

" $94x - 1052 = \pm\sqrt{685584}$

" $94x - 1052 = \pm 828$

Whence, $94x = 1880$

$\therefore \quad x = 20$ lbs. p. for 8 c., *Ans.*

SIMULTANEOUS QUADRATIC EQUATIONS.

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2. Given $x^2 + y^2 = 25$ (1)
And $x + y = 7$ (2)
Squaring (2), $x^2 + 2xy + y^2 = 49$ (3)
Subt. (1) from (3), $2xy = 24$ (4)
" (4) " (1), $x^2 - 2xy + y^2 = 1$ (5)
Extracting root, $x - y = \pm 1$ (6)
Combining (2) and (6), $x = 4$ or 3;
And $y = 3$ or 4, } *Ans.*

3. Given $x^2 + y^2 = 74$ (1)
And $x + y = 12$ (2)
Squaring (2), $x^2 + 2xy + y^2 = 144$ (3)
Subt. (1) from (3), $2xy = 70$ (4)
" (4) " (1), $x^2 - 2xy + y^2 = 4$ (5)
Extracting root, $x - y = \pm 2$ (6)
Bringing down (2), $x + y = 12$
Combining (6) and (2), $x = 7$ or 5;
And $y = 5$ or 7, } *Ans.*

4. Given $x^2 - y^2 = 28$ (1)
And $x - y = 2$ (2)
Dividing (1) by (2), $x + y = 14$ (3)
Combining (3) and (2), $x = 8$;
And $y = 6$, } *Ans.*

5. Given $x^2 + y^2 = 244$ (1)
And $y - x = 2$ (2)
Squaring (2), $y^2 - 2xy + x^2 = 4$ (3)
Subt. (3) from (1), $2xy = 240$ (4)
Adding (4) to (1), $x^2 + 2xy + y^2 = 484$ (5)
Extracting root, $x + y = \pm 22$ (6)
Bringing down (2), $-x + y = 2$
Combining (6) and (2), $x = 10$ or -12;
And $y = 12$ or -10, } *Ans.*

6. Given $3x^2 - y^2 = 251$ (1)

And $x + 4y = 38$ (2)

From (2), $x = 38 - 4y$

Squaring, $x^2 = 1444 - 304y + 16y^2$

Multiplying by 3, $3x^2 = 4332 - 912y + 48y^2$ (3)

Substituting in (1),

$4332 - 912y + 48y^2 - y^2 = 251$

Transposing, etc.,

$47y^2 - 912y = -4081$

By 3d method,

$(47)^2y^2 - 47 \times 912y + (456)^2 = -191807 + 207936$

$= 16129$

Ext. root, $47y - 456 = \pm 127$

Whence, $47y = 456 \pm 127$

$\therefore \quad y = 12\frac{19}{47}$ or 7;

$x = 10$,

} *Ans.*

From (2), using 2d value of y.

7. Given $8x^2 + 5y^2 = 728$ (1)

And $6y - x = 15$ (2)

From (2), $x = 6y - 15$ (3)

Squaring, $x^2 = 36y^2 - 180y + 225$

Multiplying by 8, $8x^2 = 288y^2 - 1440y + 1800$

Substituting in (1),

$288y^2 - 1440y + 1800 + 5y^2 = 728$

Transposing, etc.,

$293y^2 - 1440y = -1072$

By 3d method,

$(293)^2y^2 - 1440 \times 293y + (720)^2 = -314096 + 518400$

$= 204304$

Ext. root, $293y - 720 = \pm 452$

Whence, $293y = 720 \pm 452$

$\therefore \quad y = 4$ or $\frac{268}{293}$;

$x = 9$,

} *Ans.*

Dropping 2d value of y from (3).

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9. Given $x + y = 9$ (1)
And $x^3 + y^3 = 189$ (2)
Div. (2) by (1), $x^2 - xy + y^2 = 21$ (3)
Squaring (1), $x^2 + 2xy + y^2 = 81$
Subtracting, $3xy = 60$ (4)
And $xy = 20$ (5)
Subt. (5) from (3), $x^2 - 2xy + y^2 = 1$ (6)
Extracting root, $x - y = \pm 1$ (7)
Bringing down (1), $x + y = 9$
Combining (7) and (1), $x = 5$ or 4;
And $y = 4$ or 5, } *Ans.*

10. Given $x - y = 2$ (1)
And $x^3 - y^3 = 98$ (2)
Div. (2) by (1), $x^2 + xy + y^2 = 49$ (3)
Squaring (1), $x^2 - 2xy + y^2 = 4$ (4)
Subt. (4) from (3), $3xy = 45$
And $xy = 15$ (5)
Adding (5) to (3), etc., $x + y = \pm 8$ (6)
Bringing down (1), $x - y = 2$
Combining (6) and (1), $x = 5$ or -3;
And $y = 3$ or -5, } *Ans.*

11. Given $3x^2 - 7y^2 = -1$ (1)
And $4xy = 24$ (2)
From (2), $x = \frac{6}{y}$ (3)
Squaring and mult. by 3, $3x^2 = \frac{108}{y^2}$
Substituting in (1), $\frac{108}{y^2} - 7y^2 = -1$
Clearing, $108 - 7y^4 = -y^2$
Transposing, $7y^4 - y^2 = 108$
By 3d method, $y^2 = 4$ or $-3\frac{6}{7}$
Dropping negative value, $y = 2$;
From (3), $x = 3$, } *Ans.*

12. Given $x^2 - xy + y^2 = 19$ (1)
And $xy = 15$ (2)
Subt. (2) from (1), $x^2 - 2xy + y^2 = 4$ (3)
Extracting root, $x - y = \pm 2$ (4)
Adding (1) to 3 times (2),
$x^2 + 2xy + y^2 = 64$ (5)
Extracting root, $x + y = \pm 8$ (6)
Combining (6) and (4), $x = \pm 5$;
And $y = \pm 3$, } *Ans.*

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15. Given $x + y = 27$ (1)
And $xy = 180$ (2)
Squaring (1), $x^2 + 2xy + y^2 = 729$
Multiplying (2) by 4, $4xy = 720$
Subtracting, $x^2 - 2xy + y^2 = 9$ (3)
Extracting root, $x - y = \pm 3$ (4)
Bringing down (1), $x + y = 27$
Combining (4) and (1), $x = 15$ or 12;
And $y = 12$ or 15, } *Ans.*

16. Given $x - y = 14$ (1)
And $xy = 147$ (2)
Squaring (1), $x^2 - 2xy + y^2 = 196$ (3)
Multiplying (2) by 4, $4xy = 588$ (4)
Add. (4) to (3), $x^2 + 2xy + y^2 = 784$ (5)
Extracting root, $x + y = \pm 28$ (6)
Bringing down (1), $x - y = 14$
Combining (6) and (1), $x = 21$ or -7;
And $y = 7$ or -21, } *Ans.*

17. Given $x^{\frac{1}{4}} - y^{\frac{1}{4}} = 3$ (1)
And $x^{\frac{1}{4}} + y^{\frac{1}{4}} = 7$ (2)
Combining (1) and (2), $x^{\frac{1}{4}} = 5$
And $y^{\frac{1}{4}} = 2$
Involving, $x = 625$;
And $y = 16$, } *Ans.*

18. Given $x^2y^2 + x^2y^3 = 12$ (1)

And $x^2y + xy^2 = 6$ (2)

Factoring (1), $x^2y^2(x + y) = 12$ (3)

" (2), $xy(x + y) = 6$ (4)

Dividing (3) by (4), $xy = 2$ (5)

" (4) by (5), $x + y = 3$ (6)

Combining (5) and (6), $x - y = \pm 1$ (7)

Combining (6) and (7), $x = 2$ or 1;
And $y = 1$ or 2, } *Ans.*

PROBLEMS.

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1. Let $x =$ greater,

And $y =$ less.

Then $x - y = 4$ (1)

And $x^3 - y^3 = 448$ (2)

Divid. (2) by (1), $x^2 + xy + y^2 = 112$ (3)

Squaring (1), $x^2 - 2xy + y^2 = 16$ (4)

Subtracting (4) from (3), $3xy = 96$

Or $xy = 32$ (5)

Add. (3) and (5), $x^2 + 2xy + y^2 = 144$ (6)

Extracting root of (6), $x + y = \pm 12$ (7)

Combining (1) and (7), $x = 8$ or -4;
And $y = 4$ or -8, } *Ans.*

2. Let $x =$ wife's age,

Then $x + 1 =$ man's "

And $x^2 + x = 930$

By 2d method, $x = -\frac{1}{2} \pm \sqrt{930 + \frac{1}{4}} = -\frac{1}{2} \pm \sqrt{\frac{3721}{4}}$

$\therefore\ x = -\frac{1}{2} \pm \frac{61}{2} = 30$ years, wife's age;
And $x + 1 = 31$ " man's " } *Ans.*

3. Let x = greater, and y = less.

Then $(x+y)x = 180$ (1)

And $(x-y)y = 16$ (2)

From (1), $x^2 + xy = 180$

" (2), $xy - y^2 = 16$

Assuming, $x = py$

Substituting, $p^2y^2 + py^2 = 180$ (3)

And $py^2 - y^2 = 16$ (4)

From (3), $y^2 = \frac{180}{p^2+p}$ (5)

" (4), $y^2 = \frac{16}{p-1}$ (6)

Equating (5) and (6), $\frac{16}{p-1} = \frac{180}{p^2+p}$

Clearing, etc., $4p^2 + 4p = 45p - 45$

Transposing, $4p^2 - 41p = -45$

By 3d, $64p^2 - 656p + (41)^2 = -720 + 1681 = 961$

Extracting root, $8p - 41 = \pm 31$

Whence, $8p = 41 \pm 31$

Dropping 2d value, $p = 9$ (7)

From (6), $y^2 = 2$

Extracting root, $y = \pm\sqrt{2}$;

And py, or $x = 9\sqrt{2}$, } *Ans.*

4. Let x = number of rows,

And y = " " trees in each.

Then $xy = 1000$ (1)

And $x - y = 15$ (2)

Squaring (2), $x^2 - 2xy + y^2 = 225$ (3)

Multiplying (1) by 4, $4xy = 4000$ (4)

Adding, $x^2 + 2xy + y^2 = 4225$ (5)

Extracting root, $x + y = \pm 65$ (6)

Combining (6) and (2), $x - y = 15$ (2)

$\therefore \quad x = 40; \quad y = 25$, *Ans.*

5. Let x = length,
And y = breadth.
Then $xy = 960$ (1)
And $x - y = 16$ (2)
Multiplying (1) by 4, $4xy = 3840$ (3)
Squaring (2), $x^2 - 2xy + y^2 = 256$ (4)
Add. (3) to (4), $x^2 + 2xy + y^2 = 4096$ (5)
Extracting root, $x + y = \pm 64$ (6)
Combining (2) and (6), $x = 40$ yards; $y = 24$ " } *Ans.*

This problem may be solved by one unknown quantity thus:

Let x = breadth.
Then $x + 16$ = length,
And $x^2 + 16x = 960$, area.
By 2d method, $x = -8 \pm \sqrt{960 + 64}$
Reducing, $x = -8 \pm 32$
$\therefore$ $x = 24$ yds.; $x + 16 = 40$ yds., *Ans.*

6. Denote the numbers by x and y.
Then $x^2 + y^2 - (x + y) = 78$ (1)
And $xy + x + y = 39$ (2)
Adding (1) to (2) $\times$ 2,
$x^2 + 2xy + y^2 + x + y = 156$ (3)
Or $*(x + y)^2 + (x + y) = 156$
By 2d method, $x + y = -\frac{1}{2} \pm \sqrt{156 + \frac{1}{4}}$
Reducing, $x + y = -\frac{1}{2} \pm \frac{25}{2}$
Dropping 2d value, $x + y = 12$ (4)
Substituting (4) in (2), $xy = 27$ (5)
Combining (4) squared and (5) $\times$ 4,
$x^2 - 2xy + y^2 = 36$ (6)
Extracting root, $x - y = \pm 6$ (7)
Combining (4) and (7), $x = 9$ or 3; And $y = 3$ or 9, } *Ans.*

* Consider $(x + y)$ as a single quantity. (Art. 69.)

7. Let x and y denote the numbers.

Then $x^2 + y^2 + x + y = 188$ (1)

And $xy = 77$ (2)

Adding (1) to (2) × 2,

$(x + y)^2 + (x + y) = 342$ (3)

By 2d method, $x + y = -\frac{1}{2} \pm \sqrt{342 + \frac{1}{4}}$

Reducing, $x + y = 18$ (4)

Subt. (4) from (1), $x^2 + y^2 = 170$ (5)

Subtracting (2) × 2 from (5),

$x^2 - 2xy + y^2 = 16$ (6)

Extracting root, $x - y = \pm 4$ (7)

Combining (4) and (7), $x = 11$ or 7;

And $y = 7$ or 11, } *Ans.*

8. Let x = length, and y = breadth.

Then $2(x + y) = 100$ (1)

And $xy = 589$ (2)

From (1), $x + y = 50$ (3)

Comb. (2) and (3), comp. sq., extract. root, etc.,

$x - y = \pm 12$ (4)

Combining (3) and (4), $x = 31$ rods;

And $y = 19$ " } *Ans.*

9. Let x and y denote the numbers.

Then $xy = 28$ (1)

And $x^2 + y^2 = 65$ (2)

From (1), $2xy = 56$ (3)

Adding, comp. sq., etc., $x + y = \pm 11$ (4)

Subt., comp. sq., etc., $x - y = \pm 3$ (5)

Combining (4) and (5), $x = \pm 7$;

And $y = \pm 4$, } *Ans.*

10. Let x = number on side of greater square,

And y = " " " " less "

Then

And $x^2 + y^2 = 1154$ (1)

$x - y = 2$ (2)

Subt. $(2)^2$ from (1), $2xy = 1150$ (3)

Add. (1) and (3), comp. sq., etc.,

$x + y = \pm 48$ (4)

Combining (2) and (4), $x = 25$;
And $y = 23$, } *Ans.*

This problem may be solved by one unknown quantity:

Let $x^2 + (x+2)^2 = 1154$

Uniting, comp. sq., etc., $x = 23$;
And $x + 2 = 25$, } *Ans.*

11. Denote the numbers by x and y.

Then $xy = 3(x+y)$ (1)

And $x^2 + y^2 = 160$ (2)

Adding (1) $\times 2$, $x^2 + 2xy + y^2 = 160 + 6(x+y)$ (3)

Transp., etc., $(x+y)^2 - 6(x+y) = 160$

Comp. sq., 2d meth., etc., $x+y = 3 \pm 13$

Dropping negative value, $x+y = 16$ (4)

Substituting in (1), $xy = 48$ (5)

Combining (4) and (5), $x - y = \pm 8$ (6)

" (4) " (6), $x = 12$ or 4;
And $y = 4$ or 12, } *Ans.*

12. Denote the numbers by x and y.

Then $xy = 6(x-y)$ (1)

And $x^2 + y^2 = 13$ (2)

Subt. (1) $\times 2$, $x^2 - 2xy + y^2 = 13 - 12(x-y)$

Transposing, etc.,

$(x-y)^2 + 12(x-y) = 13$

Completing square, etc., $x - y = -6 \pm \sqrt{13+36}$

Dropping negative val., $x - y = 1$ (3)

Substituting in (1), $xy = 6$ (4)

Combining (2) and (4) and extracting root,

$x + y = \pm 5$ (5)

" (3) " (5), $x = 3$ or -2;
And $y = 2$ or -3, } *Ans.*

RATIO.

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3. $40 : 160 = \frac{40}{160} = \frac{1}{4}$, *Ans.*

4. $1 : 8 = \frac{1}{8}$, *Ans.*

5. $64 : 320 = \frac{64}{320} = \frac{1}{5}$, *Ans.*

6. $8a^2 : 4a = \frac{8a^2}{4a} = 2a$, *Ans.*

7. $15abc : 5ab = \frac{15abc}{5ab} = 3c$, *Ans.*

8. $500 : 50 = \frac{500}{50} = 10$, *Ans.*

9. $75 : 600 = \frac{75}{600} = \frac{1}{8}$, *Ans.*

10. $35 : 35 \times 4 = \frac{35}{35 \times 4} = \frac{1}{4}$, *Ans.*

11. $2a^2 : 4a = \frac{2a^2}{4a} = \frac{a}{2}$, *Ans.*

12. $x^2 - y^2 : x + y = \frac{x^2 - y^2}{x + y} = x - y$, *Ans.*

14. $\left\{\begin{matrix} 8 : 15 \\ 25 : 30 \end{matrix}\right\} = \frac{8}{15} \times \frac{25}{30} = \frac{4}{9}$, *Ans.*

15. $\left\{\begin{matrix} a : b \\ 2b : 3ax \end{matrix}\right\} = \frac{a}{b} \times \frac{2b}{3ax} = \frac{2}{3x}$, *Ans.*

17. $24 : 96 = \frac{24}{96} = \frac{1}{4}$, *Ans.*

18. $144 : 1728 = \frac{144}{1728} = \frac{1}{12}$, *Ans.*

19. Ratio of equality. (Art. 355.)

20. Ratio of equality.

21. $35 : 7 = 5$.
Hence, ratio of greater inequality. (Art. 356.)

22. $6 : 48 = \frac{1}{8}$. Ratio of less inequality. (Art. 357.)

23. $15 : 9 = 1\frac{2}{3}$; $38 : 19 = 2$; $\therefore\ 38 : 19 > 15 : 9$, *Ans.*

24. $8 : 25 = \frac{8}{25}$; $\sqrt{4} : \sqrt{25} = \frac{2}{5}$, or $\frac{10}{25}$, $\therefore\ \frac{8}{25} < \frac{10}{25}$, *Ans.*

25. Let $x =$ the consequent,

Then $\frac{56}{x} = 8$; $8x = 56$; $\therefore\ x = 7$, *Ans.*

26. Let $x =$ the antecedent,

Then $\frac{x}{7} = 14$; $\therefore\ x = 98$, *Ans.*

PROPORTION.

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2. Let $x =$ the first term.
Then $x : 8 :: 6 : 12$
And $12x = 48$
$\therefore\ x = 4$, *Ans.*

3. Let $x =$ the third proportional.
Then $25 : 400 :: 400 : x$
And $25x = 160000$
$\therefore\ x = 6400$, *Ans.*

4. Let $x =$ the mean proportional.
Then $9 : x :: x : 16$
And $x^2 = 144$
$\therefore\ x = 12$, *Ans.*

5. Let $x =$ the greater, and $y =$ the less.
Then $x : y :: x+y : 42$ (1)
And $x : y :: x-y : 6$ (2)
Equating (1) and (2), (Theorem 9),
$x+y : x-y :: 42 : 6$
Reducing, $x+y : x-y :: 7 : 1$
And $x + y = 7x - 7y$
Transposing, etc., $x = \frac{4y}{3}$ (3)

Substituting in (2), $\frac{4y}{3} \cdot y :: \frac{4y}{3} - y : 6$

Div. 1st couplet by y, $\frac{4}{3} : 1 :: \frac{y}{3} : 6$ (Th. 6)

Mult. antecedents by 3, $4 : 1 :: y : 6$

$\therefore \quad y = 24$, less: } *Ans.*

Substituting in (3), $x = 32$, gr., }

6. Let $x =$ one part, and $18 - x =$ the other.

Then $x^2 : (18-x)^2 :: 25 : 16$ (Th. 12)

Extracting root, $x : 18-x :: 5 : 4$

Reducing, $4x = 90 - 5x$

$\therefore \quad x = 10;$ } *Ans.*

And $18 - x = 8,$ }

7. Let $x =$ the greater, and $28 - x =$ the less.

Then $\frac{x}{28-x} : \frac{28-x}{x} :: 32 : 18$

Reducing, $9x^2 = 16(28 - x)^2$

Extracting root, (Th. 12), $3x = 4(28 - x)$

Transposing, $7x = 112$

$\therefore \quad x = 16;$ } *Ans.*

And $28 - x = 12,$ }

8. Denote the numbers by x and y.

Then $xy = 24$ (1)

And $x^3 - y^3 : (x-y)^3 :: 19 : 1$ (2)

Dividing by $x - y$,

$x^2 + xy + y^2 : x^2 - 2xy + y^2 :: 19 : 1$

By Theorem 8, $3xy : (x-y)^2 :: 18 : 1$

" " 6, $xy : (x-y)^2 :: 6 : 1$

" " 1, $6(x - y)^2 = xy$

Subst. value of xy, $6(x - y)^2 = 24$

Reducing, $x - y = 2$ (3)

Combining (1) and (3), $x + y = 10$ (4)

" (3) " (4), $x = 6;$ } *Ans.*

And $y = 4,$ }

9. Denote the numbers by x and y.

Then $x+y : x-y :: 9 : 6$ (1)

And $x-y : xy :: 1 : 12$ (2)

Reducing (1) by Theorem (6),

$x+y : x-y :: 3 : 2$

By Theorem 7, $2x : x-y :: 5 : 2$

" " 1, $4x = 5(x-y)$ (3)

Reducing (2) by Th. 1, $xy = 12(x-y)$ (4)

Dividing (3) by (4), etc., $5y = 48$

$\therefore \quad y = 9\frac{3}{5}$;

Substituting in (3), $x = 48$, } *Ans.*

10. Let $x =$ the length in rods,

And $y =$ " breadth "

Then the area, $xy = 860 \times 160$ (1)

And $x : y :: 43 : 32$ (2)

Reducing (2), $32x = 43y$

And $x = \frac{43y}{32}$ (3)

Substituting in (1), $\frac{43y^2}{32} = 860 \times 160$

Reducing, $y^2 = \frac{860 \times 160 \times 32}{43}$

Cancelling 43, $y^2 = 6400 \times 16$

Extracting root, $y = 80 \times 4 = 320$ r. breadth;

Substituting in (3), $x = 430$ r. length.

11. Let $x =$ side of one,

Then $10 + x =$ " " the other.

And $x^2 : (x+10)^2 :: 4 : 9$

By Theorem 12, $x : x+10 :: 2 : 3$

" " 1, $3x = 2x + 20$

$\therefore \quad x = 20$ rods; } *Ans.*

And $10 + x = 30$ "

12. Denote the numbers by x and y.

Then $xy = 135$ (1)

And $x^2 - y^2 : (x-y)^2 :: 4 : 1$

By Theorem 6, $x + y : x - y :: 4 : 1$

" " 7, $2x : x - y :: 5 : 1$

" " 1, $2x = 5x - 5y$

And $x = \frac{5y}{3}$ (2)

Substituting in (1), $\frac{5y^2}{3} = 135$

Reducing, $y^2 = 81$

Extracting root, $y = 9$;

Substituting in (2), $x = 15$, } *Ans.*

13. Denote the numbers by x and y.

Then $xy = 320$ (1)

And $x^2 - y^2 : (x-y)^2 :: 61 : 1$

By Theorem 6, $x^2 + xy + y^2 : x^2 - 2xy + y^2 :: 61 : 1$

" " 8, $3xy : x^2 - 2xy + y^2 :: 60 : 1$

" " 1, $3xy = 60(x^2 - 2xy + y^2)$

Dividing by 3. $20(x-y)^2 = xy$

Subst. val. of xy, $20(x-y)^2 = 320$

Dividing by 20, $(x-y)^2 = 16$

Extracting root, $x - y = 4$ (2)

Combining (1) and (2), $x + y = 36$ (3)

" (2) " (3), $x = 20$;

And $y = 16$, } *Ans.*

ARITHMETICAL PROGRESSION.

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2. $l = a \pm (n-1)d = 25 + (9-1)(-2)$
$= 25 - 16 = 9$, *Ans.*

3. $l = 12 + 14 \times 4 = 68$, *Ans.*

4. $l = 1 - 12 \times \frac{1}{2} = -5$, *Ans.*

5. $l = \frac{3}{4} + 8 \times \frac{1}{8} = 1\frac{3}{4}$, *Ans.*

6. $l = 1 - 9 \times .01 = .91$, *Ans.*

8. $l = 1 + 14 \times 3 = 43$, *Ans.*

9. $l = 31 - 8 \times 2 = 15$, *Ans.*

10. $l = 1 + 29 \times 1\frac{1}{2} = 44\frac{1}{2}$, *Ans.*

11. $l = x + 24 \times 2x = 49x$, *Ans.*

12. $l = 2a + (n - 1)\, 3a = 3an - a$, *Ans.*

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2. $s = \frac{a + l}{2} \times n$; therefore,

$$s = \frac{\frac{1}{2} + 30}{2} \times 50 = 762\frac{1}{2}, \textit{ Ans.}$$

3. $s = \frac{6 + 42}{2} \times 9 = 216$, *Ans.*

4. $s = \frac{5 + 75}{2} \times 35 = 1400$, *Ans.*

5. $s = \frac{2 + 1}{2} \times 17 = 25\frac{1}{2}$, *Ans.*

6. $l = 2 + 19 \times 3 = 59$; and

$$s = \frac{2 + 59}{2} \times 20 = 610, \textit{ Ans.}$$

7. $l = 1 + 24 \times \frac{1}{2} = 13$; and

$$s = \frac{1 + 13}{2} \times 25 = 175, \textit{ Ans.}$$

8. $l = 75 - 14 \times 3 = 33$; and

$$s = \frac{75 + 33}{2} \times 15 = 810, \textit{ Ans.}$$

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1. $l = 25 + 11 \times 3 = 58$, *Ans.*

2. $l = 58 + 44 \times 5 = 278$, *Ans.*

3. $a = l - (n - 1)\, d = 35 - 8 \times 3 = 11$, *Ans.*

4. $a = 57 - 20 \times 5 = -43$, *Ans.*

5. $d = \frac{l-a}{n-1} = \frac{85-15}{30} = 2\frac{1}{3}$, *Ans.*

6. $d = \frac{7-28}{25} = -\frac{21}{25}$, *Ans.*

7. $n = \frac{l-a}{d} + 1 = \frac{5138-23}{5} + 1 = 1024$, *Ans.*

8. $n = \frac{1152-6}{6} + 1 = 192$, *Ans.*

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1. $s = \frac{a+l}{2} \times n = \frac{9+41}{2} \times 7 = 175$, *Ans.*

2. $s = \frac{\frac{1}{3}+45}{2} \times 50 = 1130$, *Ans.*

3. $a = l - (n-1)d = 50 - 11 \times 4 = 6$, *Ans.*

4. $n = \frac{2s}{a+l} = \frac{2 \times 150}{9+41} = 6$, *Ans.*

5. $a = l + (n-1)d = 21 + 34 \times 7 = 259$, *Ans.*

6. $n = \frac{2s}{a+l} = \frac{2 \times 455}{46+24} = 13$, *Ans.*

7. $l = \frac{2s}{n} - a = \frac{2 \times 72}{9} - 27 = -11$, *Ans*

8. $l = \frac{2 \times 288}{8} - 72 = 0$, *Ans.*

9. $l = 3 + 14 \times 2 = 31$; and

$s = \frac{3+31}{2} \times 15 = 255$, *Ans.*

10. $l = 5 + 19 \times 3 = 62$, *Ans.*

11. $s = 5 + 14 \times 4 = 61$, *Ans.*

FORMULAS.

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9. Given d, n, and s, to find a, the first term.

By Formula (2), $s = \frac{a+l}{2} \times n$

Subst. for l its value in (1), $s = \frac{2a+(n-1)d}{2} \times n$

Multiplying by 2, etc., $a = \frac{2s - dn^2 + dn}{2n}$.

10. Given d, l, and s, to find a, the first term.

Equating Formulas (5) and (8),

$$\frac{l-a}{d} + 1 = \frac{2s}{a+l}$$

Clearing of fractions,

$$l^2 - a^2 + ad + dl = 2ds$$

Transposing, $a^2 - da = l^2 + dl - 2ds$

Completing sq., etc., $a = \frac{d}{2} \pm \sqrt{l^2 + dl + \frac{d^2}{4} - 2ds}$

Changing the form, $a = \frac{d}{2} \pm \sqrt{\left(l + \frac{d}{2}\right)^2 - 2ds}$

11. Given a, l, and s, to find d, the common difference.

Transposing in Formula (10), and squaring,

$$a^2 - ad = l^2 + dl - 2ds$$

Transposing, $2ds - dl - ad = l^2 - a^2$

Factoring, etc., $d = \frac{l^2 - a^2}{2s - l - a}$

12. Given l, n, and s, to find d, the common difference.

Equating (3) and (9), $l - (n-1)d = \frac{2s - dn^2 + dn}{2n}$

Mult. by $2n$, $2ln - 2dn^2 + 2dn = 2s - dn^2 + dn$

Transposing, $dn^2 - dn = 2ln - 2s$

Factoring and dividing, $d = \frac{2(ln - s)}{n(n-1)}$.

13. Given a, n, and s, to find d, the common difference.

By Formula (9), $a = \frac{2s - dn^2 + dn}{2n}$

Multiplying by $2n$, $2an = 2s - dn^2 + dn$

Transposing and dividing, $d = \frac{2s - 2an}{n^2 - n}$

14. Given d, n, and s, to find l, the last term.

By Formula (12), $d = \frac{2(ln - s)}{n(n-1)}$

Removing denom., $n(n-1)d = 2ln - 2s$

Transposing and dividing, $l = \frac{s}{n} + \frac{(n-1)d}{2}$.

15. Given a, d, and s, to find l, the last term.

By Formula (11), $d = \frac{l^2 - a^2}{2s - l - a}$

Removing denominator,

$2ds - dl - ad = l^2 - a^2$

Transposing, $l^2 + dl = 2ds - ad + a^2$

Comp. sq., etc., $l = -\frac{d}{2} \pm \sqrt{2ds + a^2 - ad + \frac{d^2}{4}}$

Or, $l = -\frac{d}{2} \pm \sqrt{2ds + \left(a - \frac{d}{2}\right)^2}$.

16. Given a, d, and s, to find n, the number of terms.

By Formula (13), $d = \frac{2s - 2an}{n^2 - n}$

Removing denom., etc.,

$dn^2 + (2a - d)n = 2s$

Completing square, etc.,

$2dn + 2a - d = \pm\sqrt{(2a-d)^2 + 8ds}$

Transposing and dividing,

$$n = \frac{\pm\sqrt{(2a-d)^2 + 8ds} - 2a + d}{2d}.$$

17. Given d, l, and s, to find n, the number of terms.

By Formula (12), $d = \frac{2(ln - s)}{n(n-1)}$

Removing denominator,

$$dn^2 - dn = 2ln - 2s$$

Transposing and factoring,

$$n^2 - \frac{2l+d}{d}n = -\frac{2s}{d}$$

Completing sq., etc., $n = \frac{2l+d}{2d} \pm \sqrt{\frac{(2l+d)^2}{4d^2} - \frac{2s}{d}}$

Or, reducing, $n = \frac{2l + d \pm \sqrt{(2l+d)^2 - 8ds}}{2d}$

18. Given a, d, and n, to find s, the sum of the terms.

By Formula (9), $a = \frac{2s - dn^2 + dn}{2n}$

Multiplying by $2n$, $2an = 2s - dn^2 + dn$

Transposing, $2s = 2an + dn^2 - dn$

$$\therefore \quad s = \frac{n}{2}[2a + (n-1)d].$$

19. Given a, d, and l, to find s, the sum of the terms.

By Formula (11), $d = \frac{l^2 - a^2}{2s - l - a}$

Remov. denom., $2ds - dl - ad = l^2 - a^2$

Transposing and factoring, $2ds = (l+a)d + l^2 - a^2$

Dividing by 2d, $s = \frac{l+a}{2} + \frac{l^2 - a^2}{2d}$.

20. Given d, l, and n, to find s, the sum of the terms.

By Formula (14), $l = \frac{s}{n} + \frac{(n-1)d}{2}$

Transposing, $\frac{s}{n} = l - \frac{(n-1)d}{2}$

Multiplying by n, $s = \frac{n}{2}[2l - (n-1)d]$

Page 212.

1. $d = \frac{31-1}{5} = 6$. Hence the series:

1, 7, 13, 19, 25, 31, *Ans.*

2. $d = \frac{48-3}{10} = 4\frac{1}{2}$. Hence the series:

3, $7\frac{1}{2}$, 12, $16\frac{1}{2}$, 21, $25\frac{1}{2}$, 30, $34\frac{1}{2}$, 39, $43\frac{1}{2}$, 48, *Ans.*

PROBLEMS.

1. $l = 5 + 14 \times 3 = 47$, *Ans.*

2. $l = 27 - 11 \times 3 = -6$, *Ans.*

3. $l = 7 + 19 \times 5 = 102$, *Ans.*

4. $d = \frac{l-a}{m+1} = \frac{60-2}{6} = 9\frac{2}{3}$.

Hence the series: 2, $11\frac{2}{3}$, $21\frac{1}{3}$, 31, $40\frac{2}{3}$, $50\frac{1}{3}$, 60, *Ans.*

5. $l = a + (n-1)d = \frac{1}{3} + 99 \times \frac{1}{3} = 33\frac{1}{3}$; and

$s = \frac{a+l}{2} \times n = \frac{\frac{1}{3} + 33\frac{1}{3}}{2} \times 100 = 1683\frac{1}{3}$, *Ans.*

6. $d = \frac{2s-2an}{n^2-n} = \frac{2 \times 18750 - 2 \times 5 \times 20}{400 - 20} = 98\frac{3}{19}$, *Ans.*

7. $l = 1 + 75 \times 2 = 151$; and

$s = \frac{1+151}{2} \times 76 = 5776$, *Ans.*

8. $l = 2 + 99 \times 2 = 200$;

$s = \frac{2+200}{2} \times 100 = 10100$, *Ans.*

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9. $d = \frac{l-a}{n-1} = \frac{47-2}{9} = 5$, *Ans.*

10. $d = \frac{l-a}{m+1} = \frac{72-6}{9} = 7\frac{1}{3}$.

Hence the series:

6, $13\frac{1}{3}$, $20\frac{2}{3}$, 28, $35\frac{1}{3}$, $42\frac{2}{3}$, 50, $57\frac{1}{3}$, $64\frac{2}{3}$, 72, *Ans.*

11. $d = \frac{l-a}{m+1} = \frac{108-12}{10} = 9.6$

Hence the series:

12, 21.6, 31.2, 40.8, 50.4, 60, 69.6, 79.2, 88.8, 98.4, 108, *Ans.*

12. $l = 100 - 14 \times 5 = 30$; and

$s = \frac{100+30}{2} \times 15 = 975$, *Ans.*

14. Let x = the second number,

And y = the common difference.

Then, $x-y+x+x+y = 15$; or $3x=15$, $x=5$. (1)

And $(x-y)^3+x^3+(x+y)^3 = 495$ (2)

Expanding and reducing, $3x^3+6xy^2 = 495$

Substituting value of x, etc., $30y^2 = 120$

$y = \pm 2$

Hence the numbers: 3, 5, and 7, *Ans.*

15. Since he has to pass over the ground twice for each marble, the problem requires us to find twice the sum of the series.

By Formula (1), $l = a + (n-1)\,d$

$l = 1 + 99 = 100$

By Formula (2), $s = \frac{a+l}{2} \times n$

$2s = \frac{2\,(1+100)\,100}{2} = 10100$ yds.

$= 5\frac{3}{4}$ miles nearly, *Ans.*

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16. By Formula (2), $2s = \frac{(1+12)2}{2} \times 12 = 156$, *Ans.*

17. By Formula (1), $l = 10 + 24 \times 20 = \$4.90$

" " (2), $s = \frac{10+490}{2} \times 25$

$\therefore \quad s = \$2.50 \times 25 = \$62.50$, *Ans.*

18. By Formula (2), $s = \frac{1 + 365}{2} \times 365$;

$s = 183 \times 365 = \$667.95$, *Ans.*

19. By Formula (2), $s = \frac{1 + 24}{2} \times 24 = 300$, *Ans.*

20. By Formula (1), $l = .06 + 19 \times .06 = \$1.20$, int.;

$\$1.00 + \$1.20 = \$2.20$, amount, *Ans.*

21. Denote the numbers by

$a - d$

a

And $a + d$

Then $3a = 120$; $a = 40$,

And $3a^2 + 2d^2 = 5600$

$\therefore \quad d = 20$

Hence, $a - d = 20$, $a = 40$, and $a + d = 60$, *Ans.*

22. Let n = No. of days the 2d travels,

Then $30 + 10n$ = " " miles " 1st "

By Formula (1), $l = 4 + (n - 1)$

$l = 3 + n$

By Formula (2), $s = \frac{4 + 3 + n}{2} \times n$ = dis. 2d goes.

Equating, $\frac{(7 + n)\, n}{2} = 30 + 10n$

Mult. by 2, $7n + n^2 = 60 + 20n$

Transpos., $n^2 - 13n = 60$

Comp. sq., etc., $n = \frac{13}{2} \pm \sqrt{60 + \frac{169}{4}}$

$\therefore \quad n = 16.61 +$ days, *Ans.*

23. Denote the numbers by

$a - 3d$, $a - d$, $a + d$, and $a + 3d$,

Square of 1st, $a^2 - 6ad + 9d^2$

" " 4th, $a^2 + 6ad + 9d^2$

Sum, $2a^2 + 18d^2 = 4500$ (1)

Square of 2d, $a^2 - 2ad + d^2$

" " 3d, $a^2 + 2ad + d^2$

Sum, $2a^2 + 2d^2 = 4100$ (2)

Subtracting (2) from (1), $16d^2 = 400$

$\therefore \quad d = 5$

Substituting in (2), $2a^2 = 4050$

$\therefore \quad a = 45$

Hence, $a - 3d = 30$, $a - d = 40$, $a + d = 50$, $a + 3d = 60$, } *Ans.*

24. Let n = No. of days B travels.

By Formula (1), $l = 7 + (n - 1)\,2$

$=$ distance A goes the last day.

$l = 5 + 2n$

By Formula (2), $s = \frac{7 + 5 + 2n}{2} \times n = 6n + n^2$,

Hence, $9 + 6n + n^2 =$ No. of miles A travels.

By Formula (1), $l = 11 + n - 1 = 10 + n$

$=$ distance B goes the last day.

By Formula (2), $s = \frac{11 + 10 + n}{2} \times n = \frac{21n + n^2}{2}$

$=$ B's distance.

Whence, $9 + 6n + n^2 = \frac{21n + n^2}{2}$

Reducing, $n = 3$ or 6 days, *Ans.*

25. By Formula (1), $l = 10 - 20 \times \frac{1}{3} = \frac{10}{3}$

" " (2), $s = \frac{10 + \frac{10}{3}}{2} \times 21 = 140$, *Ans.*

26. By Formula (1),

$l = 1 + 59 \times 3 = \$178$ last payment;

By For. (2), $s = \frac{1 + 178}{2} \times 60 = \$5370$ debt, } *Ans.*

GEOMETRICAL PROGRESSION.

Page 216.

1. $l = ar^{n-1} = 5 \times 2^5 = 160$, *Ans.*
2. $l = 2 \times 3^7 = 4374$, *Ans.*
3. $l = 72 \times (\frac{1}{2})^4 = 4\frac{1}{2}$, *Ans.*
4. $l = 5 \times 4^3 = 320$, *Ans.*
5. $l = 7 \times 2^4 = 112$, *Ans.*
6. $l = 10 \times (-5)^5 = -31250$, *Ans.*

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2. $s = \dfrac{lr - a}{r - 1} = \dfrac{2000 \times 5 - 8}{4} = 2498$, *Ans.*
3. $s = \dfrac{5000 \times 10 - 9}{9} = 5554\frac{5}{9}$, *Ans.*
4. $s = \dfrac{25000 \times 4 - 5}{3} = 33331\frac{2}{3}$, *Ans.*
5. $s = \dfrac{20 \times 6 - 15}{5} = 21$, *Ans.*
6. $s = \dfrac{12 \times 4 - 25}{3} = 7\frac{2}{3}$, *Ans.*

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1. $l = ar^{n-1} = 3 \times 10^4 = 30000$, *Ans.*
2. $l = 5 \times 5^5 = 15625$, *Ans.*
3. $a = \dfrac{l}{r^{n-1}} = \dfrac{256}{2^7} = 2$, *Ans.*
4. $a = \dfrac{243}{3^4} = 3$, *Ans.*
5. $r = \left(\dfrac{l}{a}\right)^{\frac{1}{n-1}} = \left(\dfrac{2592}{2}\right)^{\frac{1}{4}} = 6$, *Ans.*
6. $r = \left(\dfrac{2500}{4}\right)^{\frac{1}{4}} = \sqrt[4]{25 \times 25} = 5$, *Ans.*

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1. $s = \frac{lr - a}{r - 1} = \frac{108 \times 3 - 2}{2} = 161$, *Ans.*

2. $a = lr - s(r - 1) = 54 \times 3 - 80 \times 2 = 2$, *Ans.*

3. $l = \frac{a + s(r - 1)}{r} = \frac{4 + 160 \times 4}{5} = 128\frac{4}{5}$, *Ans.*

4. $r = \frac{s - a}{s - l} = \frac{15624 - 4}{15624 - 12500} = 5$, *Ans.*

5. $s = \frac{lr - a}{r - 1} = \frac{150 \times 6 - 5}{5} = 179$, *Ans.*

6. $l = \frac{a + s(r - 1)}{r} = \frac{7 + 200 \times 9}{10} = 180.7$, *Ans.*

FORMULAS.

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9. Given n, r, and s, to find a.

By Formula (1), $l = ar^{n-1}$

Multiplying by r, $lr = ar^n$

Substituting in For. (6), $a = ar^n - (r - 1)s$

Transposing, etc., $ar^n - a = (r - 1)s$

Factoring, etc., $a = \frac{(r - 1)s}{r^n - 1}$.

10. Given l, n, and s, to find a.

Equating Formulas (4) and (8),

$$\frac{s - a}{s - l} = \left(\frac{l}{a}\right)^{\frac{1}{n-1}}$$

Involving and clearing of fractions,

$$a(s - a)^{n-1} = l(s - l)^{n-1}.$$

11. Given a, n, and s, to find l.

Transp. For. (10), $l(s - l)^{n-1} = a(s - a)^{n-1}$.

12. Given n, r, and s, to find l.

Equating Formulas (3) and (9),

$$\frac{l}{r^{n-1}} = \frac{(r-1)s}{r^n - 1}$$

Multiplying by r^{n-1}, $\quad l = \dfrac{(r-1)sr^{n-1}}{r^n - 1}$.

13. Given a, l, and s, to find n.

By Formula (10),

$$a(s-a)^{n-1} = l(s-l)^{n-1}$$

By logarithms.

$$\log. a + \log. (s-a)(n-1) = \log. l + \log. (s-l)(n-1)$$

Transposing,

$$\log. (s-a)(n-1) - \log. (s-l)(n-1) = \log. l - \log. a$$

Factoring, etc.,

$$n - 1 = \frac{\log. l - \log. a}{\log. (s-a) - \log. (s-l)}$$

Transposing, $\quad n = \dfrac{\log. l - \log. a}{\log. (s-a) - \log. (s-l)} + 1.$

14. Given a, r, and s, to find n.

By Formula (9), $\quad a = \dfrac{(r-1)s}{r^n - 1}$

Removing denominator,

$$ar^n - a = (r-1)s$$

Transposing, $\quad ar^n = a + (r-1)s$

By logarithms,

$$\log. a + \log. r \times n = \log. [a + (r-1)s]$$

Transp., etc., $\quad n = \dfrac{\log. [a + (r-1)s] - \log. a}{\log. r}$.

15. Given l, r, and s, to find n.

By Formula (12), $$l = \frac{(r-1)\,sr^{n-1}}{r^n - 1}$$

Removing denominator, $$lr^n - l = (r-1)\,sr^{n-1}$$

Transposing, $$lr^n - (r-1)\,sr^{n-1} = l$$

Factoring, $$[lr - (r-1)\,s]\,r^{n-1} = l$$

Dividing, $$r^{n-1} = \frac{l}{lr - (r-1)\,s}$$

By logarithms,

$$\log. r \times (n-1) = \log. l - \log. [lr - (r-1)\,s]$$

Dividing and transposing,

$$n = \frac{\log. l - \log. [lr - (r-1)\,s]}{\log. r} + 1.$$

16. Given a, n, and s, to find r.

By Formula (9), $$a = \frac{(r-1)\,s}{r^n - 1}$$

Removing denom., $$ar^n - a = rs - s$$

Transposing, $$ar^n - rs = a - s$$

Dividing, $$r^n - \frac{s}{a}r = 1 - \frac{s}{a}.$$

17. Given l, n, and s, to find r.

By Formula (12), $$l = \frac{(r-1)\,sr^{n-1}}{r^n - 1}$$

Removing denom., $$lr^n - l = (r-1)\,sr^{n-1}$$

Transposing, $$lr^n - (r-1)\,sr^{n-1} = l$$

Or, $$lr^n - sr^n + sr^{n-1} = l$$

Factoring, etc., $$r^n + \frac{s}{l-s}r^{n-1} = \frac{l}{l-s}.$$

18. Given a, n, and r, to find s.

By Formula (9), $$a = \frac{(r-1)\,s}{r^n - 1}$$

Removing denom., $$a(r^n - 1) = (r-1)\,s$$

Dividing, $$s = \frac{a(r^n - 1)}{r-1}.$$

19. Given l, n, and r, to find s.

By Formula (12), $$l = \frac{(r-1)\,sr^{n-1}}{r^n - 1}$$

Removing denom., $$lr^n - l = (r^n - r^{n-1})\,s$$

Dividing, $$s = \frac{lr^n - l}{r^n - r^{n-1}}.$$

20. Given a, l, and n, to find s.

By Formula (4), $$r = \frac{\sqrt[n-1]{l}}{\sqrt[n-1]{a}}$$

Substituting in For. (2), $$s = \frac{\frac{l^n\sqrt[n-1]{l}}{\sqrt[n-1]{a}} - a}{\frac{\sqrt[n-1]{l}}{\sqrt[n-1]{a}} - 1}$$

Multiplying both terms by $\sqrt[n-1]{a}$,

$$s = \frac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}.$$

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2. $r = \left(\frac{l}{a}\right)^{\frac{1}{m+1}} = (128 \div \frac{1}{2})^{\frac{1}{8}} = \sqrt[8]{256} = 4.$

Ans. $\frac{1}{2}$, 2, 8, 32, 128.

PROBLEMS.

1. By Formula (2), $$s = \frac{lr - a}{r - 1}.$$

Substituting, $$s = \frac{2916 \times 3 - 6}{2} = 4371, \textit{ Ans.}$$

2. By Formula (18), $s = \frac{ar^n - a}{r-1}$, or $s = \frac{a - ar^n}{1 - r}$.

Substituting, $$s = \frac{\frac{1}{2} - \frac{1}{2} \times (\frac{1}{3})^8}{\frac{2}{3}} = \frac{\frac{1}{2} - \frac{1}{13122}}{\frac{2}{3}}$$

$$= \tfrac{1640}{2187}, \textit{ Ans.}$$

3. By Formula (1), $l = ar^{n-1} = 1 \times 3^{14}$;

" " (2), $s = \frac{lr - a}{r - 1} = \frac{3^{15} - 1}{2} = \frac{(3^5)^3 - 1}{2}$

$= \frac{(243)^3 - 1}{2} = 7174453$, *Ans.*

4. By Formula (1), $l = 1 \times (\frac{2}{3})^{11}$;

" " (2), $s = \frac{lr - a}{r - 1} = \frac{(\frac{2}{3})^{12} - 1}{-\frac{1}{3}}$

$= 2\frac{1}{177147}$, *Ans.*

5. By Formula (1), $l = ar^{n-1} = 2 \times 3^{14} = 9565938$, *Ans.*

6. By Formula (1), $l = 3 \times 3^{15} = 43046721$, *Ans.*

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8. By Formula (1), $l = ar^{n-1}$

$l = 1 \times 2^{11} = 2048$

By Formula (2), $s = \frac{lr - a}{r - 1}$

$s = \frac{2048 \times 2 - 1}{2 - 1} = \$4095$, *Ans.*

9. By For. (1), $l = 1 \times 3^9 = 19683$,

" " (2), $s = \frac{19683 \times 3 - 1}{3 - 1} = \frac{59048}{2}$

$= \$295.24$, entire cost;
$\$196.83$, last cow, } *Ans.*

10. By Formula (1), $l = 1 \times 2^9 = 512$

" " (2), $s = \frac{512 \times 2 - 1}{2 - 1} = \$10.23$, *Ans.*

11. By the conditions, $a + ar + ar^2 = 26$ (1)

And $a^2 + a^2r^2 + a^2r^4 = 364$ (2)

Transposing ar in (1) and squaring,

$a^2 + 2a^2r^2 + a^2r^4 = 676 - 52ar + a^2r^2$

Reducing, $a^2 + a^2r^2 + a^2r^4 = 676 - 52ar$ (3)

Eq. (2) and (3), $676 - 52ar = 364$

$$ar = 6 \qquad (4)$$

$$a = \frac{6}{r} \qquad (5)$$

Substituting in (1), $\frac{6}{r} + 6 + 6r = 26$

Reducing, $r^2 - \frac{10}{3}r = -1$

Completing square, etc., $r = 3$

From (5), $a = 2$

Hence, $a = 2$, $ar = 6$, $ar^2 = 18$, *Ans.*

12. By Formula (1), $l = 1 \times 2^{31} = 2^{31}$

" " (2), $s = \frac{2^{31} \times 2 - 1}{2 - 1} = 2^{32} - 1$

$= \$4294967.295$, *Ans.*

13. By the conditions, $a + ar + ar^2 = 130$

And $ar + ar^2 + ar^3 = 390$

Factoring, $a(1 + r + r^2) = 130$

" $ar(1 + r + r^2) = 390$

Dividing, $r = 3$

Substituting, $a(1 + 3 + 9) = 130$

$a = 10$, $ar = 30$, $ar^2 = 90$, $ar^3 = 270$, *Ans.*

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14. By the conditions, $a + ar + ar^2 = 210$ (1)

And $a - ar^2 = 90$ (2)

From (1), $a = \frac{210}{1 + r + r^2}$ (3)

" (2), $a = \frac{90}{1 - r^2}$ (4)

Equating (3) and (4), $\frac{210}{1 + r + r^2} = \frac{90}{1 - r^2}$

Reducing, $7 - 7r^2 = 3 + 3r + 3r^2$

Completing square, etc., $r = \frac{1}{2}$

Substituting in (4), $a = 120$

Hence, $a = \$120$, $ar = \$60$, $ar^2 = \$30$, *Ans.*

15. By the conditions, $a+ar+ar^2+ar^3 = 468$ (1)

And $ar + ar^2 + ar^3 + ar^4 = 2340$ (2)

Factoring, $a(1 + r + r^2 + r^3) = 468$ (3)

" $ar(1 + r + r^2 + r^3) = 2340$

Dividing, $r = 5$

Substituting in (3), $156a = 468$

$\therefore \quad a = 3$

Hence the numbers are 3, 15, 75, 375, 1875, *Ans.*

16. Denote the shares by $\frac{x^2}{y}$, x, y, and $\frac{y^2}{x}$.

Then $\frac{x^2}{y} + x + y + \frac{y^2}{x} = 700$ (1)

And $\frac{y^2}{x} - \frac{x^2}{y} : y-x :: 37 : 12$ (2)

Clearing (1),

$x^3+x^2y+xy^2+y^3 = 700xy$

Adding, $2x^2y + 2xy^2 = 2x^2y + 2xy^2$ (Ax. 2)

$(x + y)^3 = 700xy + 2xy(x+y)$ (3)

Reducing (2), $\frac{y^3-x^3}{xy} : y-x :: 37 : 12$

Dividing by $(y - x)$, (Theorem 6),

$\frac{y^2 + xy + x^2}{xy} : 1 :: 37 : 12$

Multiplying by xy, (Theorem 6),

$y^2 + xy + x^2 : xy :: 37 : 12$

Or, (Th. 7), $(y + x)^2 : xy :: 49 : 12$ (4)

Multiplying by $y + x$, (Theorem 6),

$(y + x)^3 : xy :: 49(y+x) : 12$

Or, $(y + x)^3 = \frac{49xy(x+y)}{12}$ (5)

Equating (3) and (5),

$700xy + 2xy(x + y) = \frac{49xy(x+y)}{12}$

Reducing, $8400 + 24(x + y) = 49(x + y)$

And $x + y = 336$ (6)

Substituting in (4), $49xy = (336)^2 \times 12$

$\therefore \quad xy = (48)^2 \times 12$

Multiplying by 4, $4xy = (48)^2$

$= 110592 \qquad (7)$

Squaring (6), $(x+y)^2 = (336)^2$

$= 112896 \qquad (8)$

Subt. (7) from (8), $(y-x)^2 = 2304$

Extracting root, $y - x = 48 \qquad (9)$

Combining (6) and (9), $\underline{y + x = 336}$

$$\left.\begin{aligned} y &= \$192; \\ x &= \$144; \\ \frac{x^2}{y} &= \$108; \\ \frac{y^2}{x} &= \$256, \end{aligned}\right\} \textit{Ans.}$$

17. By Formula (1), $ar^{n-1} = l$

Substituting, $10000r^4 = 1464.$

Extracting square root, $100r^2 = 121$

" " " $10r = 11$

$\therefore \quad r = \frac{11}{10}$, or 1.1, *Ans.*

18. Denote the numbers by $\frac{x^2}{y}$, x, y, and $\frac{y^2}{x}$.

Then $\frac{x^2}{y} + x + y + \frac{y^2}{x} = 15 \qquad (1)$

And $\frac{x^4}{y^2} + x^2 + y^2 + \frac{y^4}{x^2} = 85 \qquad (2)$

Assume $x + y = s$

" $xy = p$

Then $x^2 + y^2 = s^2 - 2p$

And $x^3 + y^3 = s^3 - 3sp$

From (1), $\frac{x^2}{y} + \frac{y^2}{x} = 15 - s \qquad (3)$

" (2), $\frac{x^4}{y^2} + \frac{y^4}{x^2} = 85 - s^2 + 2p \qquad (4)$

Squaring (3), $\frac{x^4}{y^2} + \frac{y^4}{x^2} = 225 - 30s + s^2 - 2p$ (5)

Equating (4) and (5),

$$85 - s^2 + 2p = 225 - 30s + s^2 - 2p$$

Transposing, $85 + 4p = 225 - 30s + 2s^2$ (6)

From (3), $x^3 + y^3 = 15p - sp$

Or, $s^3 - 3sp = 15p - sp$

$$p = \frac{s^3}{15 + 2s} \quad (7)$$

Substituting in (6),

$$85 + \frac{4s^3}{15 + 2s} = 225 - 30s + 2s^2$$

Removing denominator, etc.,

$$3s^2 + 17s = 210$$

Comp. sq., etc., $s = 6$ or $x + y = 6$ (8)

Substituting in (7), $p = 8$ or $xy = 8$ (9)

Comb. (8) and (9), $x - y = 2$ (10)

" (8) " (10), $x = 4$, $y = 2$,

And $\frac{x^2}{y} = 8$, and $\frac{y^2}{x} = 1$. Hence, 8, 4, 2, 1, *Ans.*

INFINITE SERIES.

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4. $1 - x \,)\, 1 + x \,(\, 1 + 2x + 2x^2 + 2x^3 + 2x^4$, etc., *Ans.*

$1 - x$

$2x$

$2x - 2x^2$

$2x^2$

$2x^2 - 2x^3$

$2x^3$

$2x^3 - 2x^4$

$2x^4$, etc.

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7. $x^2 - y^2$ $\quad x - \frac{y^2}{2x} - \frac{y^4}{8x^3} - \frac{y^6}{16x^5}$, etc., *Ans.*

x^2

$2x - \frac{y^2}{2x}$ | $-y^2$

$-y^2 + \frac{y^4}{4x^2}$

$2x - \frac{y^2}{x} - \frac{y^4}{8x^3}$ | $-\frac{y^4}{4x^2}$

$-\frac{y^4}{4x^2} + \frac{y^6}{8x^4} + \frac{y^8}{64x^6}$

$2x - \frac{y^2}{x} - \frac{y^4}{4x^3} - \frac{y^6}{16x^5}$ | $-\frac{y^6}{8x^4} - \frac{y^8}{64x^6}$, etc.

8. $1 + 1$ $\quad (1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16}$, etc., *Ans.*

1

$2 + \frac{1}{2}$) 1

$1 + \frac{1}{4}$

$2 + 1 - \frac{1}{8}$) $-\frac{1}{4}$

$-\frac{1}{4} - \frac{1}{8} + \frac{1}{64}$

$2 + 1 - \frac{1}{4} + \frac{1}{16}$) $\frac{1}{8} - \frac{1}{64}$, etc.

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10. The exponents of x decrease by 2. (Art. 271.)

The coefficients are found by means of the Binomial Formula. (Art. 270.)

1st coef. $= 1$

2d " $= n = \frac{1}{2}$

3d " $= n \times \frac{n-1}{2} = \frac{\frac{1}{2} \times -\frac{1}{2}}{2} = -\frac{1}{2 \cdot 4}$

4th " $= n \times \frac{n-1}{2} \times \frac{n-2}{3} = -\frac{1}{2 \cdot 4} \times -\frac{3}{2}$

$= \frac{3}{2 \cdot 4 \cdot 6}$

$$5\text{th coef.} = n \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4}$$

$$= \frac{3}{2\cdot4\cdot6} \times -\frac{5}{4} = -\frac{3\cdot5}{2\cdot4\cdot6\cdot8}, \text{ etc.}$$

Hence we have,

$$(x^2+y)^{\frac{1}{2}} = x + \frac{x^{-1}y}{2} - \frac{x^{-3}y^2}{2\cdot4} + \frac{3x^{-5}y^3}{2\cdot4\cdot6} - \frac{3\cdot5x^{-7}y^4}{2\cdot4\cdot6\cdot8}, \text{ etc.}$$

Or, transferring x to the denominator (Art. 279),

$$(x^2+y)^{\frac{1}{2}} = x + \frac{y}{2x} - \frac{y^2}{2\cdot4x^3} + \frac{3y^3}{2\cdot4\cdot6x^5} - \frac{3\cdot5y^4}{2\cdot4\cdot6\cdot8x^7}, \text{ etc.}$$

11. Find the first five terms as in the preceding solution.

$$6\text{th term} = n \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5}$$

$$= -\frac{3\cdot5}{2\cdot4\cdot6\cdot8} \times -\frac{7}{5} = \frac{3\cdot5\cdot7}{2\cdot4\cdot6\cdot8\cdot10}.$$

Page 231.

1. The ratio may be found by dividing any term by the preceding one.

$$s = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = 1\tfrac{1}{2}, \textit{ Ans.}$$

2. $s = \dfrac{1}{1-(-\frac{1}{2})} = \dfrac{1}{\frac{3}{2}} = \frac{2}{3}$, *Ans.*

3. $s = \dfrac{\frac{1}{2}}{1-\frac{1}{2}} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = 1$, *Ans.*

4. $s = \dfrac{\frac{1}{2}}{1-\frac{2}{3}} = \dfrac{\frac{1}{2}}{\frac{1}{3}} = 1\frac{1}{2}$, *Ans.*

5. $s = \dfrac{\frac{2}{3}}{1-\frac{2}{3}} = \dfrac{\frac{2}{3}}{\frac{1}{3}} = 2$, *Ans.*

6. $s = \dfrac{3}{1-\frac{2}{3}} = \dfrac{3}{\frac{1}{3}} = 9$, *Ans.*

7. $s = \dfrac{4}{1-\frac{2}{5}} = \dfrac{4}{\frac{2}{5}} = 10$, *Ans.*

8. $s = \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{1}{3}$, *Ans.*

9. $s = \frac{\frac{6}{10}}{1 - \frac{1}{10}} = \frac{\frac{6}{10}}{\frac{9}{10}} = \frac{2}{3}$, *Ans.*

10. $s = \frac{1}{a} \div \left(1 - \frac{1}{a}\right) = \frac{1}{a} \div \left(\frac{a-1}{a}\right) = \frac{1}{a} \times \frac{a}{a-1}$

$= \frac{1}{a-1}$, *Ans.*

11. $s = \frac{10}{1 - \frac{4}{5}} = \frac{10}{\frac{1}{5}} = 50$ rods, *Ans.*

LOGARITHMS.

Page 235.

1. Log. 7 = 0.84510, *Ans.*
2. " 9 = 0.95424, *Ans.*
3. " 108 = 2.03342, *Ans.*
4. " 176 = 2.24551, *Ans.*
5. " 1990 = 3.29885
   223 × 9 = 200
   Log. 1999 = 3.30085, *Ans.*
6. " 0.95 = $\bar{1}$.97772, *Ans.*
7. " 0.0125 = $\bar{2}$.09691, *Ans.*
8. " 0.0075 = $\bar{3}$.87506, *Ans.*
9. " 16.40 = 1.21484
   264 × 5 = 132
   Log. 16.45 = 1.21616, *Ans.*
10. " 185.0 = 2.26717
    235 × 3 = 70
    Log. 185.3 = 2.26787, *Ans.*

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11. Given 2.17231 = log. 148.7, *Ans.*

17026
301) 20500 (68

12. Given 1.25261 = log. 17.89, *Ans.*

25042
249) 21900 (87

13. Given 3.27715 = log. 1893, *Ans.*

27646
235) 69000 (293

14. Given 2.30963 = log. 204, *Ans.*

15. Given 4.29797 = log. 19858.29, *Ans.*

29667
223) 1300000 (5829

16. Given $\bar{1}$.14488 = log. 0.1396, *Ans.*

14302
322) 18600 (57

17. Given $\bar{2}$.29136 = log. 0.01956, *Ans.*

29003
223) 13300 (59

18. Given $\bar{3}$.30928 = log. 0.002038, *Ans.*

30750
212) 1780 (8

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2. Log. 109.0 = 2.03743
416 × 3 = 125
" 14.10 = 1.14922
301 × 7 = 211
3.19001 = log. 1548.86, *Ans.*

18752
281) 249000 (886

3. Log. 1.460 = .16435
301 × 5 = 151
" 1.340 = .12711
322 × 7 = 225
.29522 = log. 1.973, *Ans.*

447
223) 750 (3

4. Log. .074 = $\bar{2}$.86923
" 1500 = 3.17609
2.04532 = log. 111, *Ans.*

6. Log. 12.40 = 1.09342
349 × 8 = 279
1.09621
" 0.16 = $\bar{1}$.20412
1.89209 = log. 78, *Ans.*

7. Log. .045 = $\bar{2}$.65321
" 1.20 = .07918
$\bar{2}$.57403 = log. .0375, *Ans.*

8. Log. 1.380 = .13988
322 × 1 = 32
.14020
" .096 = $\bar{2}$.98227
1.15793 = log. 14.38, *Ans.*

534
301) 2590 (8

9. Log. — 128 = 2.10721
" — 47 = 1.67210
.43511 = log. 2.723, *Ans.*

457
158) 540 (3

10. Log. $-186 = \bar{2}.26951$
" $-0.064 = \bar{2}.80618$

$3.46333 = \log. 2906.3$, *Ans.*

240
147) 930 (63

11. Log. $-0.156 = \bar{1}.19313$
" $-0.86 = \bar{1}.93450$

$\bar{1}.25863 = \log. .1814$, *Ans.*

768
235) 950 (4

12. Log. $-0.194 = \bar{1}.28780$
" $0.042 = \bar{2}.62325$

$.66455 = \log. -4.619$, *Ans.*

370
93) 850 (9

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14. Log. $.135 = \bar{1}.13033$

4

$\bar{4}.52132 = \log. .0003321$, *Ans*

15. Log. $1.42 \times 10 = 1.5229 = \log. 33.335$, *Ans.*

.52244
130) 4600 (35

16. Log. $1.230 = .08991$
$349 \times 4 = 140$

.09131
25

45655
18262

$2.28275 = \log. 191.77$, *Ans.*

103
223) 17200 (77

18. Log. 143.0 = 2.15534
301 × 2 = 60
3) 2.15594
.71865 = log. 5.23, *Ans.*

19. Log. 1.62 = .20952
Dividing by 6, .03492 = log. 1.0836, *Ans.*
342
416) 15000 (36

20. Log. 1540 = 3.18752
281 × 9 = 253
8) 3.19005
.39875 = log. 2.504, *Ans.*
794
171) 810 (4

21. Log. 1876 ÷ 10 = .32732 = log. 2.124, *Ans.*

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23. Log. .001624 = $\bar{3}$.21058
Div. by 6, 6) $\bar{6}$+3.21058
$\bar{1}$.53509 = log. .342+, *Ans.*

24. Log. .01449 = $\bar{2}$.16107
Div. by 7, 7) $\bar{7}$+5.16107
$\bar{1}$.73729 = log. .546+, *Ans.*

25. Log. .0001236 = $\bar{4}$.09200
Divid. by 8, 8) $\bar{8}$+4.09200
$\bar{1}$.51150 = log. .324+, *Ans.*

27. Log. 1.07 × 4 = .11752
" 1500 = 3.17609
Adding, 3.29361 = log. $1966.05, *Ans.*

28. Log. 1.05×33 = .69927
" 370 = 2.56820
Adding, 3.26747 = log. $1851.27+, *Ans.*

BUSINESS FORMULAS.

Page 245.

2. $p = cr = \$4370 \times .08 = \$349.60$, *Ans.*

3. $r = \frac{p}{c} = \frac{\$500}{2500} = .20$; 20 per cent, *Ans.*

4. $r = \frac{\$300}{\$1800} = .16\frac{2}{3}$; $16\frac{2}{3}$ per cent, *Ans.* (Art. 238.)

5. $c = \frac{p}{r} = \frac{\$67.48}{.25} = \$269.92$, *Ans.*

6. $c = \frac{p}{r} = \frac{\$1575}{.12\frac{1}{2}} = \$12600$, *Ans.*

7. $s = c(1 + r) = \$750 \times 1.15 = \$862.50$, *Ans.*

8. $s = c(1 - r) = \$960 \times .87\frac{1}{2} = \$840$, *Ans.*

10. $c = \frac{s}{1 - r} = \frac{\$540}{.90} = \$600$, *Ans*

12. $t = \frac{1}{.06} = 16\frac{2}{3}$; $16\frac{2}{3}$ years, *Ans.*

13. $t = \frac{1}{.10} = 10$; 10 years, *Ans.*

14. $r = \frac{1}{t} = \frac{1}{8} = .12\frac{1}{2}$; $12\frac{1}{2}$ per cent, *Ans.*

15. $r = \frac{1}{t} = \frac{1}{40} = .02\frac{1}{2}$; $2\frac{1}{2}$ per cent, *Ans.*

17. $a = p(1 + r)^n = \$1500 \times (1.05)^6 = \$2010.14$, *Ans.*

18. $a = \$2000 \times (1.03)^6 = \$2388.05$, *Ans.*

19. $a = \$5000 \times (1.01)^8 = \$5414.28$, *Ans.*

21. $P = \frac{s}{1 + nr} = \frac{\$3600}{1.30} = \$2769.23$, pres. wr.;
$\$3600 - \$2769.23 = \$830.77$, discount, } *Ans.*

22. $P = \frac{\$7800}{1.3\sigma} = \$6000$, present worth;
$d = \$7800 - \$6000 = \$1800$, discount, } *Ans.*

24. $P = \frac{s}{(1 + r)^n} = \frac{\$2300}{(1.06)^5} = \$1718.75$, *Ans.*

26. $d = \$2500 \times .15 = \$375$;
$\$2500 - \$375 = \$2125$, cash value, *Ans.*

28. $m = \frac{c(1 + r)}{1 - d} = \frac{\$1.75 \times 1.20}{.90} = \$2.33\frac{1}{3}$, *Ans.*

29. $m = \frac{\$6.50 \times 1.25}{.92} = \$8.83+$, *Ans.*

31. $R = \frac{pr}{c} = \frac{\$6000 \times .06}{\$6000 + \$180} = 5\frac{2}{3}\frac{5}{0}\frac{5}{9}$ per cent, *Ans.*

32. $R = \frac{pr}{c} = \frac{\$1000 \times .10}{\$1000 - \$200} = 12\frac{1}{2}$ per cent, *Ans.*

33. $R = \frac{\$5000 \times .10}{\$5000 + \$200} = 9\frac{8}{1\,3}$ per cent, *Ans.*

34. $R = \frac{\$100 \times .06}{\$100} = 6$ per cent, Massachusetts;

$R = \frac{\$100 \times .08}{\$100 + \$2} = 7\frac{4}{5}\frac{2}{1}$ per cent, Ohio.

Hence the Ohio bonds are preferable, *Ans.*

36. $a = \frac{s}{1 + r} = \frac{\$25000}{1.015} = \$24630.54$, invest.;
$\$25000 - \$24630.54 = \$369.46$, commission, } *Ans.*

38. $a = \frac{(1+r)^n - 1}{r} s = \frac{(1.07)^4 - 1}{.07} \$300 = \frac{.3108}{.07} \$300$
$= \$1332$, *Ans.*

39. $a = \frac{(1.05)^{10}-1}{.05}\$500 = \frac{.629015}{.05}\$500 = \$6290.15$, *Ans.*

40. $s = a \div \frac{(1+r)^n - 1}{r} = \$5000 \div \frac{(1.05)^5 - 1}{.05}$

$= \frac{250}{.276} = \$905.80$, *Ans.*

41. $s = a \div \frac{(1+r)^n - 1}{r} = \$20000 \div \frac{(1.10)^5 - 1}{.10}$

$= \$20000 \div .61 = \$3278.69$, *Ans.*

42. $s = \$30000 \div \frac{(1.06)^{10}-1}{.06} = \frac{\$1800}{.79} = \$2278.48$, *Ans.*

43. $s = \frac{(1.07)^4-1}{.07}\$650 = \frac{.3108}{.07}\$650 = \$2886$, *Ans.*

NOTE.—The answers will vary slightly according to the number of decimals used in the solution.

44. $s = \frac{(1+r)^n - 1}{r}a = \frac{(1.06)^6 - 1}{.06}\$880 = \frac{.418}{.06}\$880$

$= \$6130.67$, *Ans.*

45. $s = \frac{(1.05)^7-1}{.05}\$340 = \frac{.407}{.05}\$340 = \$2767.60$, *Ans.*

47. $P = \frac{1-(1.04)^{-5}}{.04}\$525 = \frac{.178}{.04}\$525 = \$2336.25$, *Ans.*

49. $P = \frac{a}{r} = \frac{\$850}{.06} = \$14166.67$, *Ans.*

51. $P = \frac{a}{r}[(1+r)^{-n} - (1+r)^{-n-N}]$

$P = \frac{\$2500}{.06}[(1.06)^{-6} - (1.06)^{-18}]$

$P = \frac{\$2500 \times .3546}{.06} = \$14775$, *Ans.*

53. $a = \frac{Pr}{1-(1+r)^{-n}} = \frac{\$3840 \times .05}{1-(1.05)^{-20}}$

$= \frac{\$3840 \times .05}{.7687} = \$249.77$, *Ans.*

IMAGINARY QUANTITIES.

Pages 265, 266.

2. $+\sqrt{-x} \times -\sqrt{-y}$
$= +\sqrt{x} \times \sqrt{-1} \times -\sqrt{y} \times \sqrt{-1}$
$= -\sqrt{xy} \times -1 = +\sqrt{xy}$, *Ans.*

3. $\sqrt{-9} \times \sqrt{-4} = \sqrt{9} \times \sqrt{-1} \times \sqrt{4} \times \sqrt{-1}$
$= 6 \times -1 = -6$, *Ans.*

4. $\sqrt{-2} \times \sqrt{18} = \sqrt{2} \times \sqrt{-1} \times \sqrt{18}$
$= \sqrt{36} \times \sqrt{-1} = 6\sqrt{-1}$, *Ans.*

5. $\sqrt{-x} \times \sqrt{y} = \sqrt{x} \times \sqrt{-1} \times \sqrt{y}$
$= \sqrt{xy} \times \sqrt{-1} = \sqrt{-xy}$, *Ans.*

7. $\dfrac{\sqrt{-x}}{\sqrt{-x}} = \dfrac{\sqrt{x} \times \sqrt{-1}}{\sqrt{x} \times \sqrt{-1}} = 1$, *Ans.*

8. $\dfrac{\sqrt{-x}}{\sqrt{y}} = \dfrac{\sqrt{x} \times \sqrt{-1}}{\sqrt{y}} = \sqrt{\dfrac{-x}{y}}$, *Ans.*

9. $\dfrac{\sqrt{x}}{\sqrt{-y}} = \dfrac{\sqrt{x}}{\sqrt{y} \times \sqrt{-1}} = \sqrt{\dfrac{x}{-y}}$, *Ans.*

10. $\dfrac{10\sqrt{-14}}{2\sqrt{-7}} = \dfrac{10\sqrt{14} \times \sqrt{-1}}{2\sqrt{7} \times \sqrt{-1}} = 5\sqrt{2}$, *Ans.*

11. $\dfrac{c\sqrt{-1}}{d\sqrt{-1}} = \dfrac{c}{d}$, *Ans.*

IMPOSSIBLE PROBLEMS.

Page 268.

2. Let $x =$ the number.

Then $\frac{x}{5} - \frac{x}{4} = 15$

Clearing of fractions, $4x - 5x = 300$.

It is impossible that $4x$ should be greater than $5x$.

3. Let $x =$ one part,

And $y =$ the other.

Then $x + y = 8$ (1)

And $xy = 18$ (2)

Now the product will be greatest when the parts are equal. Making y equal to x, the equations become,

$2x = 8$ (3)

$x^2 = 18$ (4)

From (3), $x = 4$ (5)

Squaring (5), $x^2 = 16$ (6)

Equations (4) and (6) are contradictory. Hence, the problem is impossible.

NEGATIVE SOLUTIONS.

Page 269.

3. Let $x =$ the number of years.

Then $36 + x = (20 + x)\,2$

Uniting terms, $x = -4$, *Ans.*

HORNER'S METHOD.

Page 273.

3. $x^3 + 3x^2 + 5x = 178.$

A	*B*	*C*	*D*	*a b c d e*
1	+3	+5	= 178	($x = 4.5388$, *Ans.*
	4	28	132	
	7	33	46 = *D'*	*D'*+*C'*=*b*
	4	44	42.375	46+77 =.5
	11	77 = *C'*	3.625 = *D''*	
	4	7.75	2.797377	
	15 = *B'*	84.75	.827623 = *D'''*	
	.5	8	.749942	
	15.5	92.75 = *C''*	.077681	
	.5	.4959	.074994	
	16	93.2459		
	.5	.4968		
	16.5 = *B''*	93.74|2|7 = *C'''*		
	.03			
	16.53			
	.03			
	16.56			

4. $5x^3 + 9x^2 - 7x = 2200.$

A	*B*	*C*	*D*	*a b c d*
5	+9	−7	= 2200	(x=7.1073536, *Ans.*
	35	+308	2107	
	44	+301	93 = *D'*	
	35	553	86.545	
	79	854 = *C'*	6.455 = *D''*	
	35	11.45	6.144311215	
	114 = *B'*	865.45	.310688785 = *D'''*	
	.5	11.5	.263570321	
	114.5	876.95 = *C''*	.047118464	
	.5	.808745	.043928386	
	115	877.758745	.003190078	
	.5	.80899	.002635703	
	115.5 = *B''*	878.56|7|7|3|5=*C'''*	.000554375	
	.035		.000527140	
	115.535		.000027235	
	.035			
	115.57			

5. $x^3 + x^2 + x = 100$.

A	B	C	D	$abcde$
1	+1	+1	=100	(4.264429+
	4	20	84	
	5	21	16 = D'	
	4	36	11.928	
	9	57 = C'	4.072 = D''	
	4	2.64	3.788376	
	13 = B'	59.64	.283624 = D'''	
	13.2	2.68	.256071744	
	13.4	62.32 = C''	.027552256 = D^{iv}	
	13.6 = B''	.8196	.025631441984	
	13.66	63.1396	.001920814016 = D^{v}	
	13.72	8232	.001281682442	
	13.78 = B'''	63.9628 = C'''	.000639131574	
	13.784	.055136	.000576757098	
	13.788	64.017936	.000062374476	
	13.792 = B^{iv}	.055152		
	13.7924	64.073088 = C^{iv}		
	13.7928	.00551696		
		64.07860496		
		.00551712		
		64.084122|0|8 = C^{v}		

$x = 4.264429+$, *Ans.*

TEST EXAMPLES FOR REVIEW.

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1. $6a + 4a \times 5 + 8a \div 2 - 3a + 12a \times 4 = 75a$, *Ans.*

2. $(8x+3x)\,5+4x+7-(5x+9x) \div 7 = 57x+7$, *Ans.*

3. $(5ax - ab + 4cd) - (2ax - 4ab + 2cd)$
$= 3ax + 3ab + 2cd$, *Ans.*

4. $4bc + [3cd - (2xy - mn)\,5 + 3bc]$
$= 7bc + 3cd - 10xy + 5mn$, *Ans.*

5. See Book, Article 75, Prin. 4.

6. See Book, Article 91.

7. See Book, Article 91.

8. Given $\frac{2x}{3} - (x + 8) = \frac{48}{9} + \frac{12}{7} - 17\frac{5}{7}$

Uniting terms, $\frac{2x}{3} - x = \frac{16}{3} - 8$

Multiplying by 3, etc., $x = 8$, *Ans.*

9. Given $\frac{4x^2}{5} \div \frac{x}{5} + 2x = \frac{36}{3} \times \frac{31}{2}$

Reducing, $6x = \frac{36}{3} \times \frac{31}{2}$

$\therefore \quad x = 31$, *Ans.*

10. $3b^2c - 6b^2c^2 - c^2d = c(3b^2 - 6b^2c - cd)$, *Ans.*

11. $3x^2y - 9x^2z - 18x^2yz = 3x^2(y - 3z - 6yz)$, *Ans.*

12. $a^{2n} - b^{2n} = (a^n + b^n)(a^n - b^n)$, *Ans.*

13. $8a - 4 = 2 \times 2(2a - 1)$, *Ans.*

14. $a^4 - 1 = (a^2 + 1)(a + 1)(a - 1)$, *Ans.*

15. Let $x =$ one,

Then $31 - x =$ the other.

And $5x - 9(31 - x) = 1$

Reducing, $14x = 280$

$\therefore \quad x = 20;$

$31 - x = 11,$ } *Ans.*

16. See Book, Article 233.

17. Let $x =$ No. shots each fired.

Then $\frac{2}{3}x + \frac{3}{4}x = 34$

Reducing, $8x + 9x = 34 \times 12$

$\therefore \quad x = 24$ shots, *Ans.*

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18. Denote the quantities by x and y.

Then $xy = a$ (1)

$$\frac{x}{y} = b \quad (2)$$

From (2), $x = by$

Substituting in (1), $by^2 = a$

Dividing by b, $y^2 = \frac{a}{b}$

Extracting root, $y = \pm\sqrt{\frac{a}{b}}$

From (2), $y = \frac{x}{b}$

Substituting in (1), $\frac{x^2}{b} = a$

Multiplying by b, $x^2 = ab$

Extracting root, $x = \pm\sqrt{ab}$;

And $y = \pm\sqrt{\frac{a}{b}}$, } *Ans.*

19. $\frac{a^2 - 1}{ab - b} = \frac{(a+1)(a-1)}{b(a-1)} = \frac{a+1}{b}$, *Ans.*

20. $\frac{a+b}{a^2 - b^2} = \frac{1}{a-b}$, *Ans.*

21. $9x^2y^2 + 12xyz + 4z^2 = (3xy + 2z)(3xy + 2z)$, *Ans.*

22. $9b^2 - 6bc + c^2 = (3b - c)(3b - c)$, *Ans.*

23. See Book, Article 231.

24. Let $4x$ = length of fence.

Then x = No. of acres.

Reducing area to sq. rods, $x^2 = 160x$

$\therefore \quad x = 160$

And $4x = 640$ rods, *Ans.*

25. Let $2x$ = entire distance.
Then $x \div 1\frac{1}{2}$ = hours of ascent,
And $x \div 4\frac{1}{2}$ = " " descent.
By conditions, $\frac{2x}{3} + \frac{2x}{9} = 13$
Multiplying by 9, $8x = 117$
$\therefore \quad 2x = 29\frac{1}{4}$ miles, *Ans.*

26. Given $b - \frac{1+x}{1-x} = 0$
Multiplying by $1-x$, $b - bx = 1 + x$
Transposing, $bx + x = b - 1$
$\therefore \quad x = \frac{b-1}{b+1}$, *Ans.*

27. See Book, Article 103.

28. See Book, Article 104.

29. $$\frac{(x^2-y^2)(x+y)}{(x^2+2xy+y^2)(x-y)} = \frac{(x+y)(x-y)(x+y)}{(x+y)^2(x-y)}$$
$$= 1, \textit{ Ans.}$$

30. $$\frac{a^4-b^4}{(a^2-2ab+b^2)(a^2+b^2)} = \frac{(a^2+b^2)(a^2-b^2)}{(a-b)^2(a^2+b^2)}$$
$$= \frac{a+b}{a-b}, \textit{ Ans.}$$

31. Multiply the terms of the second fraction by $1-a^2$.
$$\frac{1+a^2}{1-a^4} - \frac{1-a^2}{1+a^2} = \frac{1+a^2}{1-a^4} - \frac{1-2a^2+a^4}{1-a^4} = \frac{3a^2-a^4}{1-a^4}, \textit{ Ans.}$$

32. Let x = number.
Then $x + \frac{x}{4} + \frac{x}{5} - \frac{x}{6} = 154$
Clearing of fractions,
$60x + 15x + 12x - 10x = 154 \times 60$
Uniting terms, $77x = 154 \times 60$
$\therefore \quad x = 120$, *Ans.*

33. Let x = amount each had.
Then $x - \$30 = 2(x - \$40)$
$\therefore \quad x = \$50$, *Ans.*

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34. Let x = No. of hours.
Then $24 + 8x$ = distance ship sails,
And $12x$ = " privateer.
By conditions, $12x = 24 + 8x$.
$\therefore \quad x = 6$ hours, *Ans.*

35. Given $\frac{8x + 3y}{7} = 7$ (1)

And $\frac{7y - 3x}{2} - y = 0$ (2)

Clearing of fractions, $8x + 3y = 49$ (3)
And $-3x + 5y = 0$ (4)
Mult. (3) by 5, $40x + 15y = 245$
" (4) by 3, $-9x + 15y = 0$
Subtracting, $49x = 245$
$\therefore \quad x = 5$;
Substituting 5 for x in (4), $y = 3$, } *Ans.*

36. Given $x = \frac{y - 2}{7} + 5$ (1)

And $4y - \frac{x + 10}{3} = 3$ (2)

Clearing of fractions, $7x - y = 33$ (3)
And $-x + 12y = 19$ (4)
Mult. (4) by 7, $-7x + 84y = 133$ (5)
Adding (3) and (5), $83y = 166$
$\therefore \quad y = 2$;
Substituting 2 for y in (1), $x = 5$, } *Ans.*

37. Let a = the distance; m = rate of one;
n = rate of the other;
And x = the required time.
Then $mx + nx = a$
$\therefore \quad x = \frac{a}{m + n}$. Hence, the

RULE.—*Divide the given distance by the sum of the rates; the quotient will be the time of meeting.*

38. Let $2x$ = entire distance,

Then $\frac{x}{12+4}$ = time down stream,

And $\frac{x}{12-4}$ = " up "

By conditions, $\frac{x}{16}+\frac{x}{8}=8$

Multiplying by 16, $x+2x=128$

$3x=128$

$\therefore \quad 2x=85\frac{1}{3}$ miles, *Ans.*

39. Let $\frac{x}{y}$ = the fraction,

Then $\frac{x-6}{y-6}=\frac{1}{2}$ (1)

And $\frac{x+6}{y+6}=\frac{3}{4}$ (2)

Clearing of frac., etc., $2x-y=6$ (3)

And $4x-3y=-6$ (4)

Mult. (3) by 2, $4x-2y=12$ (5)

Subtracting (4) from (5), $y=18$

Substitut. 18 for y in (3), $x=12$

Hence, $\frac{x}{y}=\frac{12}{18}$, *Ans.*

40. Let x = the greater,

y = " less.

Then $x+y : y :: 8 : 3$ (1)

And $x^2-y^2=49$ (2)

From (1), $x=\frac{5y}{3}$ (3)

Sub. value of x in (2), $\frac{25y^2}{9}-y^2=49$ (4)

Multiplying by 9, $16y^2=441$

Extracting root, $4y=21$

$\therefore \quad y=\frac{21}{4}=5\frac{1}{4}$ less;

Sub. $\frac{21}{4}$ for y in (3), $x=\frac{35}{4}=8\frac{3}{4}$ gr., } *Ans.*

41. Given $10x + 6y = 76$ (1)

$4y - 2z = 8$ (2)

And $6x + 8z = 88$ (3)

Multiplying (2) by (4), $16y - 8z = 32$ (4)

Adding (3) and (4), $6x + 16y = 120$ (5)

Multiplying (5) by 5, $30x + 80y = 600$ (6)

" (1) by 3, $30x + 18y = 228$

Subtracting, $62y = 372$

$\therefore \quad y = 6;$

Substituting 6 for y in (1), $x = 4;$

" 6 " y " (2), $z = 8,$ } *Ans.*

42. Given $2x + 3y + z = 24$ (1)

$3x + y + 2z = 26$ (2)

And $x + 2y + 3z = 34$ (3)

Multiplying (1) by 3, $6x + 9y + 3z = 72$ (4)

" (2) by 2, $6x + 2y + 4z = 52$ (5)

Subtracting (5) from (4), $7y - z = 20$ (6)

Multiplying (3) by 3, $3x + 6y + 9z = 102$ (7)

Subtracting (2) from (7), $5y + 7z = 76$ (8)

Multiplying (6) by 7, $49y - 7z = 140$

Adding, $54y = 216$

$\therefore \quad y = 4;$

Substituting 4 for y in (6), $z = 8;$

" these values in (1), $x = 2,$ } *Ans.*

43. Let $x =$ A's money,

$y =$ B's "

And $z =$ C's "

Then $x + y + z = \$180$ (1)

$x - y + z = \$60$ (2)

And $x + y - z = \frac{z}{4}$ (3)

Subtracting (2) from (1), $2y = \$120$

$\therefore \quad y = \$60$

Adding (1) and (2), $2x + 2z = \$240$ (4)

" (2) " (3), $2x - \frac{z}{4} = \$60$

Subtracting, $\frac{9z}{4} = \$180$

$\therefore \quad z = \$80$, C's;
Subst. \$80 for z in (4), $x = \$40$, A's; } *Ans.*
$y = \$60$, B's,

44. Let $x =$ circumference of fore-wheel,
Then $x + 3 =$ " " hind "

And $\frac{240}{x} = \frac{240}{x+3} + 40$

Clearing of fractions, etc.,

$x^2 + 3x = 18$

Comp. sq., etc, $x = 3$ meters, circ. f. wheel;
And $x + 3 = 6$ " " h. " } *Ans.*

45. Let $x =$ side of one,
Then $x + 2 =$ " " the other.
Difference of cubes,

$6x^2 + 12x + 8 = 488$

Reducing, $x^2 + 2x = 80$

Comp. sq., etc., $x = 8$ ft., side of one;
And $x + 2 = 10$ " " other, } *Ans.*

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46. See Book, Article 232.

47. Let $7x =$ one part,
And $11x =$ the other.
Then $18x = 126$

$\therefore \quad x = 7$

$7x = 49$;
$11x = 77$, } *Ans.*

48. Let x = number of meters.

Then $$\frac{\$120}{x} + .50 = \frac{\$120}{x - 8}$$

Dividing by .50, clearing of fractions, and reducing,

$$x^2 - 8x = 1920$$

Comp. square, etc., $x = 48$ meters, *Ans.*

49, See Book, Article 472.

$$s = c(1 + r) = \$175 \times 1.25 = \$218.75;$$

$$\$218.75 \div 89 = \$2.457, \textit{ Ans.}$$

50. See Book, Article 490.

51. Let x = price of horse,

Then $x + \$100$ = " carriage.

And $x + 100 : x :: x : 50$

Changing to an equation,

$$x^2 - 50x = 5000$$

Comp. square, etc., $x = \$100$, horse;
And $x + \$100 = \$200$, carriage, } *Ans.*

52. See Book, Article 247.

53. Let x = share of younger,

Then $\underline{x + 35}$ = " elder.

Adding, $2x + 35 = 165$

Transp., etc., $x = 65$ hectares, younger;
$x + 35 = 100$ " elder, } *Ans.*

54. Let x = the number.

Then $$3x - 40 = 51 - \frac{x}{2}$$

Transposing, etc., $7x = 182$

$\therefore$ $x = 26$, *Ans.*

55. Let x = price of a sheep.
y = " " lamb.

Then $6x + 7y = \$71$ (1)

$4x + 8y = \$64$ (2)

Mult. (2) by 6, $24x + 48y = \$384$

" (1) by 4, $24x + 28y = \$284$

Subtracting, $20y = \$100$

$\therefore y = \$5$;

Substituting 5 for y in (2), $x = \$6$, } *Ans.*

56. Let x = No. who voted for one,

Then $x + 271$ = " " " the other.

And $2x + 271 = 1425$

Transposing, $2x = 1154$

$\therefore x = 577$ for one;

And $x + 271 = 848$ " the other, } *Ans.*

57. Let x = C's age,

Then $3x$ = B's "

And $6x$ = A's "

Adding, $10x = 150$

$\therefore x = 15$ years, C's age;

$3x = 45$ " B's " } *Ans.*

$6x = 90$ " A's "

58. $\sqrt{243} = \sqrt{81 \times 3} = 9\sqrt{3}$, *Ans.*

59. $\sqrt{y^2 + ay^2} = \sqrt{y^2(1 + a)} = y\sqrt{1+a}$, *Ans.*

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60. $x^{\frac{3}{4}} = (x^4)^{\frac{1}{4}}$; $y^{\frac{1}{3}} = (y^3)^{\frac{1}{3}}$, *Ans.*

61. $3(a - b) = \sqrt[3]{27(a - b)^3}$, *Ans.*

62. Let x = price per bushel.

Then $17x - 13x = \$3.60$

Or, $4x = \$3.60$

$\therefore x = \$0.90$, *Ans.*

63. Let x = amount lost on second,
Then $x + 8$ = " " first.
And $2x + 8 = 3x - 2$
Transposing, x = \$10 on the second; $x + 8$ = \$18 " first, } *Ans.*

64. Let x = men first employed.
Then, by conditions, $6x : 10(x+12) :: \frac{1}{4} : \frac{3}{4}$
Changing to an equation, etc., $8x = 120$
$\therefore \quad x = 15$ men, *Ans.*

65. Let x = number in the party.
Then $8x$ = amount they paid.
And $8x = 7(x + 4)$
Reducing, $x = 28$, *Ans.*

66. Let x = wages of a woman.
And y = " " boy.
Then $8x + 6y = 72$ (1)
$6x + 11y = 80$ (2)
Mult. (2) by 4, $24x + 44y = 320$
" (1) by 3, $24x + 18y = 216$
Subtracting, $26y = 104$
$\therefore \quad y =$ \$4, boy; } *Ans.*
Substituting 4 for y in (1), $x =$ \$6, woman, }

67. $\sqrt{153x} = \sqrt{9 \times 17x} = 3\sqrt{17x}$. Hence, 9, *Ans.*

68. Given $\sqrt{x + 12} = \sqrt{a + 12}$
Squaring, etc., $x = a$, *Ans.*

69. Given $\frac{\sqrt{y}}{y} = \frac{y - ay}{\sqrt{y}}$
Multiplying by $\sqrt{y}$, etc., $1 = y - ay$
Factoring and dividing, $y = \frac{1}{1 - a}$, *Ans.*

5. See Book, Article 75, Prin. 4.

6. See Book, Article 91.

7. See Book, Article 91.

8. Given $\frac{2x}{3} - (x+8) = \frac{48}{9} + \frac{12}{7} - 17\frac{5}{7}$

Uniting terms, $\frac{2x}{3} - x = \frac{16}{3} - 8$

Multiplying by 3, etc., $x = 8$, *Ans.*

9. Given $\frac{4x^2}{5} \div \frac{x}{5} + 2x = \frac{36}{3} \times \frac{31}{2}$

Reducing, $6x = \frac{36}{3} \times \frac{31}{2}$

$\therefore \quad x = 31$, *Ans.*

10. $3b^2c - 6b^2c^2 - c^2d = c(3b^2 - 6b^2c - cd)$, *Ans.*

11. $3x^2y - 9x^2z - 18x^2yz = 3x^2(y - 3z - 6yz)$, *Ans.*

12. $a^{2n} - b^{2n} = (a^n + b^n)(a^n - b^n)$, *Ans.*

13. $8a - 4 = 2 \times 2(2a - 1)$, *Ans.*

14. $a^4 - 1 = (a^2 + 1)(a + 1)(a - 1)$, *Ans.*

15. Let $x =$ one,

Then $31 - x =$ the other.

And $5x - 9(31 - x) = 1$

Reducing, $14x = 280$

$\therefore \quad x = 20$;
$31 - x = 11$, } *Ans.*

16. See Book, Article 233.

17. Let $x =$ No. shots each fired.

Then $\frac{2}{3}x + \frac{3}{4}x = 34$

Reducing, $8x + 9x = 34 \times 12$

$\therefore \quad x = 24$ shots, *Ans.*

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18. Denote the quantities by x and y.

Then $xy = a$ (1)

$$\frac{x}{y} = b \quad (2)$$

From (2), $x = by$

Substituting in (1), $by^2 = a$

Dividing by b, $y^2 = \frac{a}{b}$

Extracting root, $y = \pm\sqrt{\frac{a}{b}}$

From (2), $y = \frac{x}{b}$

Substituting in (1), $\frac{x^2}{b} = a$

Multiplying by b, $x^2 = ab$

Extracting root, $x = \pm\sqrt{ab}$;

And $y = \pm\sqrt{\frac{a}{b}}$, } *Ans.*

19. $\frac{a^2 - 1}{ab - b} = \frac{(a+1)(a-1)}{b(a-1)} = \frac{a+1}{b}$, *Ans.*

20. $\frac{a+b}{a^2 - b^2} = \frac{1}{a-b}$, *Ans.*

21. $9x^2y^2 + 12xyz + 4z^2 = (3xy + 2z)(3xy + 2z)$, *Ans.*

22. $9b^2 - 6bc + c^2 = (3b - c)(3b - c)$, *Ans.*

23. See Book, Article 231.

24. Let $4x$ = length of fence.

Then x = No. of acres.

Reducing area to sq. rods, $x^2 = 160x$

$\therefore \quad x = 160$

And $4x = 640$ rods, *Ans.*

78. Let x = number of sheep.

Then $\frac{\$120}{x}$ = price per head.

And $\frac{\$120}{x} = \frac{\$120}{x+6} + \$1$

Clearing of fractions,

$$120x + 720 = 120x + x^2 + 6x$$

Transposing, $x^2 + 6x = 720$

Comp. sq., $x^2 + 6x + 9 = 720 + 9$

Ext. root, etc., $x = -3 \pm 27$

$\therefore$ $x = 24$ sheep;

And $\frac{120}{24} = \$5$, each, } *Ans.*

79. Let $10x + y$ = number.

Then $10x + y = 9(x + y)$ (1)

And $10x + y - 63 = 10y + x$ (2)

From (1), $x = 8y$ (3)

" (2), $9x - 9y = 63$ (4)

Dividing (4) by 9, $x - y = 7$

Substituting, $8y - y = 7$

$\therefore$ $y = 1$

From (3), $x = 8$

Hence, $10x + y = 81$, *Ans.*

80. Let $3x$ = distance one goes,

Then $7x$ = " the other goes.

And $10x = 150$ miles.

$\therefore$ $x = 15$ "

$3x = 45$ miles, one;

$7x = 105$ " other, } *Ans.*

81. Let x = No. of B's acres,

Then $x + 10$ = " A's "

And $\frac{\$2800}{x}$ = price per acre of B's,

" $\frac{\$2800}{x+10}$ = " " " A's

" $\frac{2800}{x} = \frac{2800}{x+10} + 5$

Clear. of frac., $2800x + 28000 = 2800x + 5x^2 + 50x$

Transposing, etc., $x^2 + 10x = 5600$

Completing square, etc., $x = -5 \pm \sqrt{5600 + 25}$

Reducing, $x = -5 \pm 75$

$\therefore$ $x = 70$ acr. B;

And $x + 10 = 80$ " A, } *Ans.*

82. $(\sqrt{x} + \sqrt{7})(\sqrt{x} - \sqrt{7}) = x - 7.$

Hence, $\sqrt{x} - \sqrt{7}$, *Ans.*

83. $(\sqrt{3x} - \sqrt{3y})(\sqrt{3x} + \sqrt{3y}) = 3x - 3y.$

Hence, $\sqrt{3x} + \sqrt{3y}$, *Ans.*

84. Given $\sqrt{b^2 + \sqrt{x}} = \frac{d+3}{\sqrt{b^2 + \sqrt{x}}}$

Multiply. by denom., $b^2 + \sqrt{x} = d + 3$

Transposing, $\sqrt{x} = d + 3 - b^2$

Squaring, $x = d^2 + 6d - 2b^2d + 9 - 6b^2 + b^4$, *Ans.*

85. Let x = clerk's salary,

Then $10x$ = mayor's "

And $11x = \$13200$

$\therefore$ $x = \$1200$, clerk;

And $10x = \$12000$, mayor, } *Ans.*

86. Denote the numbers by x and y.

Then $x+y : x-y :: 8 : 6$

And $x-y : xy :: 1 : 36$

By Theorem 1, $x = 7y$ (1)

And $xy = 36x - 36y$ (2)

Sub. $7y$ for x in (2), $7y^2 = 252y - 36y$

Dividing by y, $7y = 216$ (3)

$\therefore y = 30\frac{6}{7}$;

Equating (1) and (3), $x = 216$, } *Ans.*

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87. Denote the numbers by x and y.

Then $xy = 48$ (1)

And $x^3 - y^3 : (x-y)^3 :: 37 : 1$ (2)

By Th. 6, $x^3+xy+y^2 : x^2-2xy+y^2 :: 37 : 1$

By Theorem 8, $3xy : (x-y)^2 :: 36 : 1$

" " 6, $xy : (x-y)^2 :: 12 : 1$

Value of xy from (1), $48 : (x-y)^2 :: 12 : 1$

By Theorem 6, $4 : (x-y)^2 :: 1 : 1$

" " 12, $2 : x-y :: 1 : 1$

" " 1, $x-y = 2$ (3)

Combining (1) and (3), $x+y = 14$ (4)

" (3) " (4), $x = 8$; $y = 6$, *Ans.*

88. Let x = price per dozen; $\frac{x}{12}$ = price of one,

And $12 \div \frac{x}{12} = \frac{144}{x}$ = No. for 12 cents.

By conditions, $\frac{144}{x} = \frac{144}{x+1} + 2$ (1)

Dividing (1) by 2, $\frac{72}{x} = \frac{72}{x+1} + 1$ (2)

Clearing of fractions, $72x + 72 = 72x + x^2 + x$

Transposing, $x^2 + x = 72$

Completing square, etc., $x = -\frac{1}{2} \pm \sqrt{\frac{289}{4}}$

Reducing, $x = -\frac{1}{2} \pm \frac{17}{2} = 8$ cents, *Ans.*

89. Let x = No. of days they travel,
Then $8x$ = distance one goes,
And $7x$ = " other "
By the problem, $15x$ = 150 miles,
$\therefore$ x = 10 days, *Ans.*

90. Let x = A's income,
Then $3x$ = B's "
And $4x$ = \$1876
$\therefore$ x = \$469, A; } *Ans.*
And $3x$ = \$1407, B, }

91. Let x = cost of cow,
Then $4x$ = " horse,
And $5x$ = \$250
$\therefore$ x = \$50, cow; } *Ans.*
And $4x$ = \$200, horse, }

92. Let x = rate per hour he rode,
Then $\frac{24}{x}$ = hours spent in riding,
And $\frac{24}{3} = 8$ = " " walking.
By the problem,

$$\frac{24}{x} + 8 = 12$$

$\therefore$ x = 6 miles per hour, *Ans.*

93. Let x = the length,
And y = " width.

Then	$2x + 2y = 320$	(1)
And	$xy = 6000$	(2)
Dividing (1) by 2,	$x + y = 160$	(3)
Squaring (3),	$x^2 + 2xy + y^2 = 25600$	(4)
Mult. (2) by 4,	$4xy = 24000$	(5)
Subtracting,	$x^2 - 2xy + y^2 = 1600$	(6)
Extracting root,	$x - y = 40$	(7)
Combining (3) and (7),	x = 100 ft. length; }	*Ans.*
And	y = 60 ft. width, }	

94. $d = \frac{l-a}{m+1} = \frac{31-3}{6} = 4\frac{2}{3}$. Hence the series:

3, $7\frac{2}{3}$, $12\frac{1}{3}$, 17, $21\frac{2}{3}$, $26\frac{1}{3}$, 31, *Ans.*

95. $l = a + (n-1)\,d = \frac{1}{2} + 49 \times \frac{1}{2} = 25$;

$s = \frac{a+l}{2} \times n = \frac{\frac{1}{2}+25}{2} \times 50 = 637\frac{1}{2}$, *Ans.*

96. Let x = No. pair bought; $x-5$ = No. pair sold;

And $\frac{\$100}{x}$ = price paid a pair,

" $\frac{\$135}{x-5}$ = " received a pair.

By the problem, $\frac{100}{x} = \frac{135}{x-5} - 1$

Reducing, etc., $x = 50$ pair, *Ans.*

97. Let x = one; y = other.

Then $x : y :: 7 : 9$ (1)

And $y^2 - x^2 = 128$ (2)

Changing (1) to an equation, $9x = 7y$

$\therefore \quad x = \frac{7y}{9}$ (3)

Substituting in (2), $y^2 - \frac{49y^2}{81} = 128$

Multiplying by 81, $81y^2 - 49y^2 = 128 \times 81$

$\therefore \quad y^2 = 4 \times 81$

Extracting root, $y = 18$;

Substituting 18 for y in (3), $x = 14$, } *Ans.*

98. Let x = No. in the height,

Then $x + 43$ = " " " length.

And $x^2 + 43x = 2400$

Comp. sq., etc., $x = -\frac{43}{2} \pm \sqrt{2400 + \frac{1849}{4}}$

Reducing, $x = -\frac{43}{2} \pm \frac{107}{2}$

$\therefore \quad x = 32$, No. in height;

And $x + 43 = 75$, " " length, } *Ans.*

99. Let x = Bertha's age,
Then $3x$ = Mother's age.
And $x + 20 = (3x + 20)\frac{3}{5}$
Reducing, $5x + 100 = 9x + 60$
$\therefore$ $x = 10$ years, B.; $3x = 30$ " M., } *Ans.*

100. Let x = price paid for car,
Then $\frac{x}{15}$ = what each would have paid,
And $\frac{x}{12}$ = " " did pay.
By the problem, $\frac{x}{12} = \frac{x}{15} + \$1.75$
$\therefore$ $x = \$105$, *Ans.*

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101. See Book, Article 247,
$t = ah = \frac{12}{11} \times 8 = \frac{96}{11} = 8\frac{8}{11}$ hours $= 8{:}43\frac{7}{11}$ o'clock, *Ans.*

102. Let x = A's pages; y = B's pages.
Then $x + y = 570$ (1)
And $3x + 5y = 2350$ (2)
Mult. (1) by 3, $3x + 3y = 1710$ (3)
Subt. (3) from (2), $2y = 640$
$\therefore$ $y = 320$, B's pages; } *Ans.*
From (1), $x = 250$, A's " }

103. Let x = No. of days it will last the man,
Then $\frac{1}{x}$ = what the man drinks in 1 day,
$\frac{1}{30}$ = " wife " 1 day.
By the problem, $\frac{12}{x} + \frac{12}{30} = 1$
Reducing, etc., $x = 20$ days, *Ans.*

104. See book, Article 476.
$$t = \frac{1}{r} = \frac{1}{.07} = 14\frac{3}{7} \text{ years, } \textit{Ans.}$$

105. Let x = sum in the purse.

Then $3+2 : 7+2 :: x : x+24$

Or $5 : 9 :: x : x+24$

By Theorem 1, $5x + 120 = 9x$

$\therefore \quad x = \$30$, *Ans.*

106. Let x = whole stock,

And $\frac{2}{3}x - 200$ = A's stock.

Then $x : \frac{2}{3}x - 200 :: 3000 : 1600$

By Theorem 6, $x : \frac{5x - 1800}{9} :: 30 : 16$

By Theorem 1, $16x = \left(\frac{5x - 1800}{3}\right) 10$

Multiplying by 3, $48x = 50x - 18000$

$\therefore \quad x = \$9000$, whole stock;

And $\frac{2}{3}x - 200 = \$4800$, A's " } *Ans.*

$9000 - 4800 = \$4200$, B's "

107. Let x = No. votes for one,

Then $x+1$ = " " the other.

And $2x + 1 = 369$

$\therefore \quad x = 184$ votes for one;

And $x + 1 = 185$ " " the other, } *Ans.*

108. Let x = time by one; y = time by the other.

Then $\frac{1}{x}$ = part one does in 1 day,

$\frac{1}{y}$ = " the other does in 1 day.

Since both together can do the whole work in 16 days, they do $\frac{1}{4}$ of it in 4 days and $\frac{1}{16}$ of it in 1 day. Hence,

By the conditions, $\frac{1}{x} + \frac{1}{y} = \frac{1}{16}$ (1)

And $\frac{36}{y} = \frac{3}{4}$ (2)

Dividing by 3, etc., $y = 48$ days.

Subst. 48 for y in (1), $\frac{1}{x} + \frac{1}{48} = \frac{1}{16}$

$\therefore \quad x = 24$ days, one; $y = 48$ days, other, *Ans.*

109. Let x = digit in tens' place,
And y = " " units' "
Then $10x + y$ = number.

By the conditions, $\frac{10x+y}{xy} = 2\frac{1}{2}$ (1)

And $10x + y + 9 = 10y + x$ (2)

From (1), $40x + 4y = 9xy$ (3)

Uniting and dividing (2) by 9,

$$x + 1 = y \quad (4)$$

Substituting value of y in (3),

$$40x + 4x + 4 = 9x^2 + 9x$$

Transposing, $9x^2 - 35x = 4$

Dividing by 9, $x^2 - \frac{35}{9}x = \frac{4}{9}$

Completing square, etc., $x = \frac{35}{18} \pm \sqrt{\frac{4}{9} + \frac{1225}{18^2}}$

Reducing, $x = \frac{35}{18} \pm \sqrt{\frac{1369}{18^2}}$

And $x = \frac{35}{18} \pm \frac{37}{18}$

$\therefore$ $x = 4$

From (4), $y = 5$

Hence, $10x + y = 45$, *Ans.*

110. $r = \left(\frac{l}{a}\right)^{\frac{1}{m+1}} = \left(\frac{486}{2}\right)^{\frac{1}{5}} = \sqrt[5]{243} = 3.$

Hence the series, 2, 6, 18, 54, 162, 486, *Ans.*

111. Let x = No. of hats.

Then $\frac{80}{x}$ = price paid apiece.

And $\frac{80}{x} = \frac{80}{x+4} + 1$

Clearing of fractions, $80x + 320 = 80x + x^2 + 4x$

Transposing, $x^2 + 4x = 320$

Completing square, etc., $x = -2 \pm \sqrt{320+4}$

Reducing, $x = -2 \pm 18$

$\therefore$ $x = 16$ hats, *Ans.*

112. $l = ar^{n-1} = 2 \times 5^{11} = 97656250$, *Ans.*

113. Let x = length of shortest.
Then $5x + 3x + x = 90$ feet.
Uniting terms, $9x = 90$ "
$\therefore\ x = 10$ feet;
$3x = 30$ "
And $5x = 50$ "
} *Ans.*

114. Let $3x + 19 = 3d$,
Then $2x = 2d$,
$4x + 11 = 1st.$
And $9x + 30 = 219$
Transposing, $9x = 189$
$\therefore\ x = 21$
$4x + 11 = 95$, 1st;
$2x = 42$, 2d;
And $3x + 19 = 82$, 3d,
} *Ans.*

115. Denote the numbers by x, $\sqrt{xy}$, and y.
Then $x + \sqrt{xy} + y = 14$ (1)
And $x^2 + xy + y^2 = 84$ (2)
Transposing and squaring (1),
$x^2 + 2xy + y^2 = 196 - 28\sqrt{xy} + xy$ (3)
Subt. (2) from (3), $xy = 112 - 28\sqrt{xy} + xy$
Transposing, etc., $\sqrt{xy} = 4$ (4)
Involving, $xy = 16$
And $3xy = 48$ (5)
Subtracting (5) from (2),
$x^2 - 2xy + y^2 = 36$
Extracting root, $x - y = 6$ (6)
Subst. (4) in (1), $x + y = 10$ (7)
Comb. (6) and (7), $x = 8$
$y = 2$
$\sqrt{xy} = 4$
Hence the numbers, 8, 4, and 2, *Ans.*

116. Let x = time the coffee would last the wife.

The man drinks 1 lb. in 4 weeks.

His wife " 1 " " x "

Hence he " $\frac{1}{4}$ " " 1 week.

And she " $\frac{1}{x}$ " " 1 "

He " $\frac{3}{4}$ " " 3 weeks.

She " $\frac{3}{x}$ " " 3 "

And by the problem, $\frac{3}{4} + \frac{3}{x} = 1$

Clearing of fractions, $3x + 12 = 4x$

Transposing, $x = 12$ weeks, *Ans.*

117. Let x = sum in 1st purse,

y = " " 2d "

Then $x + y = \$300$ (1)

And $x - 30 = y + 30$ (2)

From (2), $x = y + 60$ (3)

Subst. in (1), $y + 60 + y = 300$

Transposing, $2y = 240$

$\therefore \quad y = \$120$, 2d;

From (3), $x = \$180$, 1st, } *Ans.*

118. Let x = cost of the cloth.

Then $\frac{x}{100}$ = rate per cent.

$x \times \frac{x}{100}$ = percentage gained.

And $x + \frac{x^2}{100} = 39$

Clearing of fractions,

$x^2 + 100x = 3900$

Comp. sq., etc., $x = -50 \pm \sqrt{3900 + 2500}$

Reducing, $x = -50 \pm 80$

$\therefore \quad x = \$30$, *Ans.*

119. Let x = No. of acres in one part,
And y = " " the other part.
Then $x + y = 100$ (1)
And $20x + 30y = 2450$ (2)
Mult. (1) by 20, $20x + 20y = 2000$
Subtracting, etc., $y = 45$ acres;
From (1), $x = 55$ " } *Ans.*

120. $l = ar^{n-1} = 2 \times 3^{14}$;
$$s = \frac{lr - a}{r - 1} = \frac{2 \times 3^{15} - 2}{2} = 3^{15} - 1$$
$$= 14348906, \textit{ Ans.}$$

121. Let x = No. of pine,
And y = " " hemlock.
Then $x + y = 300$
$x^2 : y^2 :: 25 : 49$
Product means = product extremes,
$49x^2 = 25y^2$
Extracting sq. root, $7x = 5y$
$$x = \frac{5y}{7}$$
Subst. in (1), $\frac{5y}{7} + y = 300$
Mult. by 7, etc., $12y = 2100$
$\therefore$ $y = 175$, hemlock;
From (1), $x = 125$, pine, } *Ans.*

122. Let x = No. of men.
By conditions, $2x = 3\frac{1}{3}(x - 150)$
Reducing, $3x = 5x - 750$
Transposing, $2x = 750$
$\therefore$ $x = 375$ men, *Ans.*

APPENDIX.

CUBE ROOT.

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1. Given.

2. $a^3+3a^2b+3ab^2+b^3$ ($a+b$, *Ans.*

a^3

$3a^2$) $3a^2b+3ab^2+b^3$

$3a^2+3ab+b^2$) $3a^2b+3ab^2+b^3$

3. $x^3+6x^2+12x+8$ ($x+2$, *Ans.*

x^3

$3x^2$) $6x^2+12x+8$

$3x^2+6x+4$) $6x^2+12x+8$

4. $x^3-6x^2y+12xy^2-8y^3$ ($x-2y$, *Ans.*

x^3

$3x^2$) $-6x^2y+12xy^2-8y^3$

$3x^2-6xy+4y^2$) $-6x^2y+12xy^2-8y^3$

5. $8a^3-48a^2+96a-64$ ($2a-4$, *Ans.*

$8a^3$

$12a^2$) $-48a^2+96a-64$

$12a^2-24a+16$) $-48a^2+96a-64$

6. $27a^3-54a^2x+36ax^2-8x^3$ ($3a-2x$, *Ans.*

$27a^3$

$27a^2$) $-54a^2x+36ax^2-8x^3$

$27a^2-18ax+4x^2$) $-54a^2x+36ax^2-8x^3$

(7.)

$a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1$ ($a^2 - 2a + 1$, *Ans.*

a^6 ($3a^4$ = First Trial Divisor.

$-6a^5 + 12a^4 - 8a^3$ ($3a^4 - 6a^3 + 4a^2$ = Complete Divisor.

$3a^4 - 12a^3 + 15a^2 - 6a + 1$ ($3a^4 - 12a^3 + 12a^2$ = Second Trial Divisor.

$3a^4 - 12a^3 + 15a^2 - 6a + 1$ ($3a^4 - 12a^3 + 15a^2 - 6a + 1$ = Complete Divisor.

(8.)

$x^9 - 3x^8 + 8x^6 - 6x^5 - 6x^4 + 8x^3 - 3x + 1$) $x^3 - x^2 - x + 1$, *Ans.*

x^9

$-3x^8 + 0 + 8x^6 - 6x^5 - 6x^4 + 8x^3 - 3x + 1$) $3x^6$ = First Trial Divisor.

$-3x^8 + 3x^7 - x^6$) $3x^6 - 3x^5 + x^4$ = Complete Divisor.

$-3x^7 + 9x^6 - 6x^5 - 6x^4 + 8x^3 - 3x + 1$) $3x^6 - 6x^5 + 3x^4$ = Second Trial Divisor.

$-3x^7 + 6x^6 - 3x^4 - x^3$) $3x^6 - 6x^5 + 3x^3 + x^3$ = Complete Divisor.

$3x^6 - 6x^5 - 3x^4 + 9x^3 - 3x + 1$) $3x^6 - 6x^5 - 3x^4 + 6x^3 + 3x^3$ = Third Trial Divisor.

$3x^6 - 6x^5 - 3x^4 + 9x^3 - 3x + 1$) $3x^6 - 6x^5 - 3x^4 + 9x^3 - 3x + 1$ = Complete Divisor.

FACTORING.

Page 284.

1. $x^2 - 9x + 20 = (x - 4)(x - 5)$, *Ans.*

Extract the square root of the first term for the first term of the factors; resolve the third term into two factors, whose sum is the coefficient of the second term, and these will be the second terms of the required factors. Thus, $\sqrt{x^2} = x$; $20 = (-5) \times (-4)$; $-5 - 4 = -9$.

2. $a^2 + 7a - 18 = (a + 9)(a - 2)$, *Ans.*
$\sqrt{a^2} = a$; $-18 = -2 \times 9$; $9 - 2 = 7$.

3. $a^2 - 13a + 40 = (a - 8)(a - 5)$, *Ans.*
$\sqrt{a^2} = a$; $40 = (-8)(-5)$; $-8 - 5 = -13$.

4. $2abc^2 - 14abc - 60ab = 2ab(c^2 - 7c - 30)$
$= 2ab(c - 10)(c + 3)$, *Ans.*
$\sqrt{c^2} = c$; $-30 = -10 \times 3$; $-10 + 3 = -7$.

5. $x^2y^2 - 2xy + 1 = (xy - 1)(xy - 1)$, *Ans.*
$\sqrt{x^2y^2} = xy$; $1 = (-1)(-1)$; $-1 - 1 = -2$.

6. $8x^2 - 32y^2 = 8(x^2 - 4y^2)$ (Art. 131)
$= 8(x + 2y)(x - 2y)$, *Ans.*

7. $x^2 + y^2z + ymz = x^2 + y(y + m)z$, *Ans.*

NOTE.—It sometimes occurs, as in this example, that only a portion of a polynomial can be factored when there is no factor common to all the terms.

8. $12a^2x - 8a^2y + 4az = 4a(3ax - 2ay + z)$, *Ans.*

9. Extract the cube root and thus find one of the three equal factors:

$$(a - x)\quad (a - x)\quad (a - x),\ \textit{Ans.}$$

10. $1 - a^4 = (1 + a^2)(1 + a)(1 - a)$, *Ans.* (Art. 131.)

11. $1 + 2a \;)\; 1 + 8a^3 \;(\; 1 - 2a + 4a^2$ (Art. 132.)

$$\begin{array}{r} 1 + 2a \\ \hline - 2a \\ - 2a - 4a^2 \\ \hline 4a^2 + 8a^3 \\ 4a^2 + 8a^3 \\ \hline \end{array}$$

Hence $(1 + 2a)(1 - 2a + 4a^2)$, *Ans.*

12. $a^6 - b^4x^2 = (a^3 + b^2x)(a^3 - b^2x)$, *Ans.* (Art. 131.)

G. C. D. OF POLYNOMIALS.

Page 284.

1. $4a^2 - 4ax - 15x^2 \mid 6a^2 + 7ax - 3x^2 =$ 1st Divisor.

Multiply by 3 $\mid$ 2 $=$ 1st Quotient.

1st Div. $= 12a^2 - 12ax - 45x^2$

$12a^2 + 14ax - 6x^2 \mid 6a^2 + 7ax - 3x^2 =$ 2d Divid.

Cancel., $-13x) - 26ax - 39x^2 \mid 6a^2 + 9ax$

2d Divisor $= 2a + 3x$ $\mid -2ax - 3x^2$

2d Quotient $= 3a - x$ $\mid -2ax - 3x^2$

$2a + 3x$, *Ans.*

2. $4x^2z^2$, *Ans.*

3. $16x^2 - y^2 = (4x + y)(4x - y)$;
$16x^2 - 8xy + y^2 = (4x - y)^2$;
Hence $4x - y$, *Ans.*

4. 1st Div. $= 6a^2 + 11ax + 3x^2 \mid 6a^2 + 7ax - 3x^2 =$ 1st Divisor.

$6a^2 + 7ax - 3x^2 \mid 1 =$ 1st Quotient.

Canceling. $2x \;)\; 4ax + 6x^2 \mid 6a^2 + 7ax - 3x^2 =$ 2d Dividend.

2d Divisor $= 2a + 3x$ $\mid 6a^2 + 9ax$

2d Quotient $= 3a - x$ $\mid -2ax - 3x^2$

$\mid -2ax - 3x^2$

$2a + 3x$, *Ans.*

5. $a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$; (Art. 131.)
$a^5 - b^2a^3 = a^3(a^2 - b^2) = a^3(a + b)(a - b)$.
Hence $(a + b)(a - b) = a^2 - b^2$, *Ans.*
(Art. 140, Prin. 1.)

6. $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$; (Art. 130.)
$x^4 - a^4 = (x^2 + a^2)(x + a)(x - a)$; (Art. 131.)
Hence $x - a$, *Ans.*

L. C. M. OF POLYNOMIALS.

Page 284.

1. $6a^2 - 4a = 2a(3a - 2)$, prime factors;
$4a^3 + 2a = 2a(2a^2 + 1)$, " "
$6a^2 + 4a = 2a(3a + 2)$, " "
$2a(2a^2+1)(3a+2)(3a-2) = 36a^5+2a^3-8a$, *Ans.*

2. $4(1 + a^2) = 2^2(1 + a^2)$;
$4(1 - a^2) = 2^2(1 + a)(1 - a)$;
$8(1 - a) = 2^3(1 - a)$; $8(1 + a) = 2^3(1 + a)$.
Hence $2^3(1 + a^2)(1 + a)(1 - a) = 8(1-a^4)$, *Ans.*

3. $a^3 - 2a + 1 = (a - 1)^2$;
$a^4 - 1 = (a^2 + 1)(a + 1)(a - 1)$;
$a^2 + 2a + 1 = (a + 1)^2$.
Hence $(a+1)^2 \times (a-1)^2 \times (a^2+1) = (a^2 - 1)^2 \times (a^2 + 1)$
$= (a^2 - 1)(a^4 - 1)$
$= a^6 - a^4 - a^2 + 1$, *Ans.*

4. $12(ab^2 - b^3) = 2^2 \times 3b^2(a - b)$;
$4(a^2 + ab) = 2^2a(a + b)$;
$18(a^2 - b^2) = 2 \times 3^2(a + b)(a - b)$.
Hence $2^2 \times 3^2ab^2(a+b)(a-b) = 36ab^2(a^2-b^2)$, *Ans.*

5. $4a^2 - 1 = (2a + 1)(2a - 1)$.
The other two quantities are prime.
Hence $(2a + 1)(2a - 1)(4a^2 + 1) = 16a^4 - 1$, *Ans.*

6. See example 2.

FRACTIONS.

Page 285.

1. $5x + \frac{x-2}{3} = 5x + \frac{5x^2 - 10x}{15x}$

$4x - \frac{3x-3}{5x} = 4x - \frac{9x-9}{15x}$

Adding, $9x + \frac{5x^2 - 19x + 9}{15x}$, *Ans.*

2. $\frac{1}{a^2 - b^2} + \frac{2ab}{a^4 - b^4}$; (Art. 101.)

$\frac{a^2+b^2+2ab}{a^4-b^4} = \frac{(a+b)^2}{a^4-b^4} = \frac{a+b}{a^3-a^2b+ab^2-b^3}$, *Ans.*

3. $3 + 5 + 7 - \frac{3a - 2x}{x} + \frac{2a}{x} + \frac{x-a}{a}$;

$15 - \frac{3a^2 - 2ax - 2a^2 - x^2 + ax}{ax}$;

$15 - \frac{a^2 - ax - x^2}{ax}$, *Ans.*

4. $\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a+ac}{ac}$;

$\frac{ac - bc + ab - ac + bc - ab + abc}{abc} = \frac{abc}{abc}$;

And $\frac{abc}{abc} = 1$, *Ans.*

5. From $a + 3h - \frac{d-b}{2}$

Take $3a - h + \frac{d+b}{3}$

$-2a + 4h + \frac{b-5d}{6}$,

Or $\frac{-12a + 24h + b - 5d}{6}$, *Ans.*

6. From $5x + \frac{x}{b}$

Take $2x - \frac{x-b}{c}$

$3x + \frac{x}{b} + \frac{x-b}{c}$,

Or $3x + \frac{cx + bx - b^2}{bc}$, *Ans.*

7. From $a + x + \frac{x}{x^2 - y^2}$

Take $a - x + \frac{1}{x+y}$

$2x + \frac{x}{x^2 - y^2} - \frac{x-y}{x^2 - y^2}$,

Or $2x + \frac{y}{x^2 - y^2}$, *Ans.*

Multiply the terms of the 2d fraction by $x - y$.

8. $\frac{a-1}{a} - \frac{a}{a-1}$;

$\frac{a^2 - 2a + 1 - a^2}{a^2 - a}$,

Or $\frac{1 - 2a}{a^2 - a}$, *Ans.*

9. $\frac{1}{1-x} = \frac{1+x}{1-x^2}$; (Art. 183.)

$\frac{1+x}{1-x^2} - \frac{2}{1-x^2} = \frac{x-1}{1-x^2}$,

Or, $\frac{x-1}{1-x^2} = -\frac{1-x}{(1+x)(1-x)} = -\frac{1}{1+x}$, *Ans.*

MULTIPLICATION OF FRACTIONS.

Page 285.

1. $1 - \frac{a-b}{a+b} = \frac{a+b-a+b}{a+b} = \frac{2b}{a+b}$;

$2 + \frac{2b}{a-b} = \frac{2a-2b+2b}{a-b} = \frac{2a}{a-b}$;

$\frac{2b}{a+b} \times \frac{2a}{a-b} = \frac{4ab}{a^2-b^2}$, *Ans.*

2.
$$\frac{\frac{4a}{3x} + \frac{3x}{2b}}{\frac{2b}{3x} + \frac{3x}{4a}}$$

$$\frac{\frac{8ab}{9x^2} + 1}{+1 + \frac{9x^2}{8ab}}$$

$$\frac{8ab}{9x^2} + 2 + \frac{9x^2}{8ab}, \text{ Ans.}$$

3. $\frac{x^2-2xy+y^2}{1} \times \frac{x+y}{x-y} = (x-y)(x+y)$

$= x^2 - y^2$, *Ans.*

Cancel $x - y$ from numerator and denominator.

4. $\frac{7b}{2a^2-8a^4} \times \frac{2a^2-4a^3}{21b}$

$$= \frac{7b}{2a^2(1+2a)(1-2a)} \times \frac{2a^2(1-2a)}{21b}.$$

Canceling common factors, etc., we have $\frac{1}{3+6a}$, *Ans.*

5. $\frac{a^2}{x+y} \times \frac{x^2-y^2}{ab} = \frac{a^2}{x+y} \times \frac{(x+y)(x-y)}{ab}$

$= \frac{a(x-y)}{b}$, *Ans.*

6. $\frac{a^2-1}{1} \times \frac{5b}{a-1} = \frac{(a+1)(a-1)}{1} \times \frac{5b}{a-1}$

$= 5b(a+1)$, *Ans.*

7. $\frac{15a-30}{2a} \times \frac{3a^2}{5a-10} = \frac{3(5a-10)}{2a} \times \frac{3a^2}{5a-10}$

$= \frac{9a}{2}$, *Ans.*

8. $y - \frac{xy}{x+y} \times y + \frac{xy}{x-y}$

$\frac{xy+y^2-xy}{x+y} \times \frac{xy-y^2+xy}{x-y}$

$\frac{y^2}{x+y} \times \frac{2xy-y^2}{x-y} = \frac{2xy^3-y^4}{x^2-y^2}$, *Ans.*

DIVISION OF FRACTIONS.

Page 285.

1. $a + \frac{2a}{a-3} = \frac{a^2-3a+2a}{a-3} = \frac{a^2-a}{a-3}$;

$a - \frac{2a}{a-3} = \frac{a^2-3a-2a}{a-3} = \frac{a^2-5a}{a-3}$;

$a + \frac{2a}{a-3} \div a - \frac{2a}{a-3} = \frac{a^2-a}{a-3} \times \frac{a-3}{a^2-5a}$

$= \frac{a-1}{a-5}$, *Ans.*

2. $\frac{1}{x} + \frac{1}{xy^3} = \frac{y^3+1}{xy^3}$; $y - 1 + \frac{1}{y} = \frac{y^2-y+1}{y}$;

$\frac{1}{x} + \frac{1}{xy^3} \div y - 1 + \frac{1}{y} = \frac{y^3+1}{xy^3} \times \frac{y}{y^2-y+1}$

$= \frac{y+1}{xy^2}$, *Ans.*

NOTE. $y^3 + 1 = (y+1)(y^2-y+1)$. (Art 129, Prin. 3.)

3. $1+\frac{1}{x}\div 1-\frac{1}{x^2}=\frac{x+1}{x}\div\frac{x^2-1}{x^2}$

$$=\frac{x+1}{x}\times\frac{x^2}{x^2-1}=\frac{x}{x-1},\ \textit{Ans.}$$

4. $\frac{3y}{2y-2}\div\frac{2y}{y-1}=\frac{3y}{2(y-1)}\times\frac{y-1}{2y}=\frac{3}{4},\ \textit{Ans.}$

5. $\frac{ab+b^2}{a^3-b^3}\div\frac{b}{a-b}=\frac{b(a+b)}{(a-b)(a^2+ab+b^2)}\times\frac{a-b}{b}$

$$=\frac{a+b}{a^2+ab+b^2},\ \textit{Ans.}$$

6. $\frac{1}{1+a}+\frac{a}{1-a}=\frac{1-a+a+a^2}{1-a^2}=\frac{1+a^2}{1-a^2}$;

$$\left(\frac{1}{1+a}+\frac{a}{1-a}\right)\div\frac{(1+a^2)^2}{(1-a^2)^2}=\frac{1+a^2}{1-a^2}\times\frac{(1-a^2)^2}{(1+a^2)^2}$$

$$=\frac{1-a^2}{1+a^2},\ \textit{Ans.}$$

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7. $\frac{\frac{3a}{2a-2}}{\frac{2a}{a-1}}=\frac{3a}{2a-2}\times\frac{a-1}{2a}=\frac{3}{4},\ \textit{Ans.}$

8. $\frac{x+\frac{x}{3}}{y+\frac{y}{4}}=\frac{3x+x}{3}\div\frac{4y+y}{4}=\frac{4x}{3}\times\frac{4}{5y}=\frac{16x}{15y},\ \textit{Ans.}$

9. $\frac{\frac{x^2}{x^2-y^2}-1}{\frac{y^2}{x^2-y^2}+1}=\frac{x^2-x^2+y^2}{x^2-y^2}\div\frac{y^2+x^2-y^2}{x^2-y^2}$

$$=\frac{y^2}{x^2-y^2}\times\frac{x^2-y^2}{x^2}=\frac{y^2}{x^2},\ \textit{Ans.}$$

10. $$\frac{\frac{1}{a}+\frac{1}{ab^3}}{b-1+\frac{1}{b}} = \frac{b^3+1}{ab^3} \div \frac{b^2-b+1}{b}$$

$$= \frac{(b+1)(b^2-b+1)}{ab^3} \times \frac{b}{b^2-b+1}$$

$$= \frac{b+1}{ab^2}, \textit{ Ans.}$$

EQUATIONS.

Page 286.

1. Let x = price of house,
Then $\$850 - x$ = " " barn.
And $5x = 12 \times \$850 - 12x$
Transposing, $17x = 12 \times \$850$
Dividing, $x = 12 \times \$50 = \$600$, house;
And $\$850 - x = \$250$, barn, } *Ans.*

2. Let $12x$ = No. of C's acres,
Then $8x$ = " A's "
And $9x$ = " B's "
Then $29x = 145$
Dividing, $x = 5$
$\therefore$ $8x = 40$ acres, A's;
$9x = 45$ " B's; } *Ans.*
$12x = 60$ " C's,

3. Let $6x$ = No. liters cask holds,
Then $2x$ = " " in it before leakage,
$\therefore$ $x = 21$ liters,
$6x = 126$ " *Ans.*

4. Given $3x - \frac{x-4}{4} - 4 = \frac{5x+14}{3} - \frac{1}{12}$

Clearing of fractions,

$$36x - 3x + 12 - 48 = 20x + 56 - 1$$

Uniting terms, $13x = 91$

$$\therefore \quad x = 7, \textit{ Ans.}$$

5. Given $\frac{3x+9}{2} = \frac{7x+5}{3} - \frac{16+4x}{5} + 6$

Clearing of fractions,

$$45x + 135 = 70x + 50 - 96 - 24x + 180$$

Uniting terms, $x = 1$, *Ans.*

6. Given $\frac{x+8}{4} - \frac{x-6}{3} + x = x + 2$

Cancel x and clear of fractions,

$$3x + 24 - 4x + 24 = 24$$

Uniting terms, $x = 24$, *Ans.*

7. Given $x - 2 = x + \frac{x+8}{4} - \frac{x-6}{3}$

Cancel x and clear of fractions,

$$-24 = 3x + 24 - 4x + 24$$

Uniting terms, $x = 72$, *Ans.*

8. Given $\frac{2x+1}{3} - \left(\frac{-x-3}{4}\right) = x$

Clearing of fractions,

$$8x + 4 + 3x + 9 = 12x$$

Uniting terms, $x = 13$, *Ans.*

9. $t = \frac{12}{11} \times 7 = \frac{84}{11} = 7{:}38\frac{2}{11}$ o'clock P. M., *Ans.*

See Art. 247.

10. Let $27x + £200 =$ stock.
$\underline{\qquad\qquad 50 =}$ annual expense.
$27x + £150$
$\underline{9x + \quad 50 =}$ gain 1st year.
$36x + £200 =$ sum " "
$\underline{\qquad\qquad 50 =}$ expense.
$36x + £150$
$\underline{12x + \quad 50 =}$ gain.
$48x + £200 =$ sum 2d year.
$64x + £200 =$ " 3d "
By conditions, $64x + £200 = 54x + £400$
Transposing, $10x = £200$
$\therefore \quad x = £20$
And $27x + £200 = £740$, *Ans.*

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1. Given $\frac{4x + 6y}{3} = 26$ (1)

And $\frac{6x}{2} - \frac{6y}{3} = 0$ (2)

From (1), $4x + 6y = 78$ (3)
" (2), $9x = 6y$ (4)
Substituting in (3), $4x + 9x = 78$
$\therefore \quad x = 6;$
And from (4), $y = 9,$ } *Ans.*

2. Given $\frac{x}{3} + \frac{y}{2} = 8$ (1)

And $\frac{x}{2} + \frac{y}{3} = 7$ (2)

Multiply (1) by 18, $6x + 9y = 144$ (3)
" (2) by 12, $\underline{6x + 4y = \quad 84}$ (4)
Subtracting, $5y = \quad 60$
$\therefore \quad y = \quad 12;$
Substituting in (4), $x = \quad 6,$ } *Ans.*

3. Given $\frac{3x}{4} - \frac{y}{2} = 9$ (1)

And $\frac{2x - 2y}{4} = 4$ (2)

Multiply each by 4, $3x - 2y = 36$ (3)

And $2x - 2y = 16$ (4)

Subtracting, $x = 20$;

Substituting in (4), $y = 12$, } *Ans.*

4. Given $\frac{x}{2} + \frac{y}{3} = b$ (1)

And $\frac{x}{3} + \frac{y}{4} = c$ (2)

Multiply (2) by 36, $12x + 9y = 36c$

" (1) by 24, $12x + 8y = 24b$

Subtracting, $y = 36c - 24b$;

Substituting in (1), $x = 18b - 24c$, } *Ans.*

5. Let x = Price paid for harness,

Then $4x$ = " " " horse,

And $3x$ = " " " buggy.

Hence, $8x = \$400$

$\therefore \quad x = \$50$, harness;

$4x = \$200$, horse; } *Ans.*

$3x = \$150$, buggy,

6. Let $10x + y$ = The Number.

Then $10x + y + (10y + x) = 121$ (1)

And $10x + y - (10y + x) = 9$ (2)

From (1), $11x + 11y = 121$

Or $x + y = 11$ (3)

From (2), $x - y = 1$ (4)

Combining (3) and (4), $x = 6$

And $y = 5$

Hence, $10x + y = 65$, *Ans.*

7. Given $x + y - z = 0$ (1)

And $x + z - y = 2$ (2)

And $y + z - x = 4$ (3)

Adding, $x + y + z = 6$ (4)

Subtracting (3) from (4), $2x = 2$

" (2) " (4), $2y = 4$

" (1) " (4), $2z = 6$

And $\left.\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}\right\}$ *Ans.*

8. Given $\frac{x}{b} + \frac{y}{c} = 1$ (1)

And $\frac{x}{b} + \frac{z}{d} = 1$ (2)

And $\frac{y}{c} + \frac{z}{d} = 1$ (3)

Adding, $\frac{2x}{b} + \frac{2y}{c} + \frac{2z}{d} = 3$ (4)

Subtracting 2 times (3), $\frac{2y}{c} + \frac{2z}{d} = 2$

$\frac{2x}{b} = 1$

Subtracting 2 times (2) from (4), $\frac{2y}{c} = 1$

" 2 " (1) " (4), $\frac{2z}{d} = 1$

And $\left.\begin{aligned} x &= \frac{b}{2}; \\ y &= \frac{c}{2}; \\ z &= \frac{d}{2}, \end{aligned}\right\}$ *Ans.*

9. Given
$$w + x + y = 6 \quad (1)$$
$$w + x + z = 9 \quad (2)$$
$$w + y + z = 8 \quad (3)$$
$$x + y + z = 7 \quad (4)$$

Adding, $3w + 3x + 3y + 3z = 30$

Or $w + x + y + z = 10 \quad (5)$

Subtracting (4) from (5), $w = 3$

" (3) " (5), $x = 2$

" (2) " (5), $y = 1$

" (1) " (5), $z = 4$

Ans.

10. Given $xy = 600$, or $y = \frac{600}{x} \quad (1)$

$$xz = 300, \text{ or } z = \frac{300}{x} \quad (2)$$
$$yz = 200 \quad (3)$$

Substituting in (3) the values of y and z,
$$yz = \frac{600}{x} \times \frac{300}{x} = 200$$

Reducing, $x^2 = 900$

Extracting root, $x = 30$

From (1), $y = 20$

From (2), $z = 10$

Ans.

11. Given
$$x + \frac{y}{4} - \frac{z}{8} = 19 \quad (1)$$
$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 22 \quad (2)$$
$$\frac{x}{4} + y + \frac{z}{2} = 33 \quad (3)$$

Multiplying (1) by 16, $16x + 4y - 2z = 304 \quad (4)$

" (2) " 12, $6x + 4y + 3z = 264 \quad (5)$

" (3) " 4, $x + 4y + 2z = 132 \quad (6)$

Subtracting (5) from (4), $10x - 5z = 40 \quad (7)$

" (6) " (5), $5x + z = 132 \quad (8)$

Multiplying (8) by (5), $25x + 5z = 660 \quad (9)$

Add (7) and (9), $35x = 700 \quad \therefore \quad x = 20$

From (8), $z = 32$

" (3), $y = 12$

Ans.

12. Given $w + 50 = x$, or $x - w = 50$ (1)
$x + 120 = 3y$, or $3y - x = 120$ (2)
$y + 120 = 2z$, or $2z - y = 120$ (3)
$z + 195 = 3w$, or $3w - z = 195$ (4)
Add (1) and (2), $3y - w = 170$ (5)
(4) × 2 + (3), $6w - y = 510$ (6)
(6) × 3 + (5), $18w - w = 1530 + 170$
Reducing, $17w = 1700$
$\therefore w = 100$
Substituting value of w in (1), $x = 150$
" " w in (5), $y = 90$
" " w in (4), $z = 105$
} *Ans.*

13. Let x, y, and z represent the ages of A, B, and C.
Then $x + 3y + 3z = 470$ (1)
And $4x + y + 4z = 580$ (2)
And $5x + 5y + z = 630$ (3)
Add (1) and (2), $5x + 4y + 7z = 1050$ (4)
Subtracting (3) from (4), $-y + 6z = 420$ (5)
Multiplying (1) by 4, $4x + 12y + 12z = 1880$ (6)
Subtracting (2) from (6), $11y + 8z = 1300$ (7)
Multiplying (5) by 11, $-11y + 66z = 4620$ (8)
Add (7) and (8), $74z = 5920$
$\therefore z = 80$ yrs., C's age;
From (5), $y = 60$ yrs., B's age;
From (1), $x = 50$ yrs., A's age,
} *Ans.*

14. Let x, y, and z be the numbers.
Then $x + y + z = 59$
And $\frac{x - y}{2} = 5$, or $x - y = 10$ (1)
And $\frac{x - z}{2} = 9$, or $x - z = 18$ (2)
Adding and reducing, $x = 29$, 1st;
Substituting in (1), $y = 19$, 2d;
" (2), $z = 11$, 3d,
} *Ans.*

GENERALIZATION.

Page 288.

1. $x = \frac{cn}{p - c} = \frac{5 \times 6}{8 - 5} = \frac{30}{3} = 10$ hours, *Ans.*

GENERAL PROBLEM.—Given two objects starting at different times and moving at different rates of speed, to find the time required by one object to overtake the other.

RULE.—Multiply the rate of the first by the number of hours, or periods of time, between starting, and divide the product by the difference between the rates.

2. $x = \frac{abc}{ab + ac + bc} = \frac{2 \times 5 \times 10}{10 + 20 + 50} = 1\frac{1}{4}$ days, *Ans.*

GENERAL PROBLEM.—Given the times required for n forces separately to produce a result, to find the time required by the united forces to produce the same result.

RULE.—Divide the product of the numbers denoting the time required by each force, by the sum of the different products of the same numbers taken $n - 1$ in a set.

3. Formula $x = \frac{n}{s}$;

$$\left.\begin{aligned} ax &= \frac{an}{s} = \frac{5 \times 4400}{22} = \$1000, \text{ A's;} \\ bx &= \frac{bn}{s} = \frac{7 \times 4400}{22} = \$1400, \text{ B's;} \\ cx &= \frac{cn}{s} = \frac{10 \times 4400}{22} = \$2000, \text{ C's,} \end{aligned}\right\} \textit{Ans.}$$

GENERAL PROBLEM.—The proportions being given, to divide a number into proportional parts.

RULE.—Divide the number by the sum of the proportions and multiply the quotient by each of the proportions in turn; the several products are the parts required.

4. Formulas,

$$F = \frac{am\,(n-1)}{m-n} = \frac{9 \times 9\,(3-1)}{9-3} = 27 \text{ years};$$
$$S = \frac{a\,(n-1)}{m-n} = \frac{9 \times 2}{6} = 3 \text{ years.}$$
Ans.

GENERAL PROBLEM.—F is now m times older than S; in a periods F will be n times older than S: to find the age of each.

RULE—1. Divide the product of the given interval, first multiple, and second multiple less one, by the difference between the multiples; the quotient is the age of the older.

2. Divide the product of the given interval and the second multiple less one, by the difference between the multiples; the quotient is the age of the younger.

EXPANDING POWERS OF BINOMIALS.

1. Powers of $2a$, $8a^3 + 4a^2 + 2a + 1$
" " $-3b$, $1 - 3b + 9b^2 - 27b^3$
Coefficients, 1 3 3 1
$(2a - 3b)^3 = 8a^3 - 36a^2b + 54ab^2 - 27b^3$, *Ans.*

2. Powers of $3x$, $81x^4 + 27x^3 + 9x^2 + 3x + 1$
" " $2y$, $1 + 2y + 4y^2 + 8y^3 + 16y^4$
Coefficients, 1 4 6 4 1
$(3x + 2y)^4 = 81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$, *Ans.*

3. Powers of 1, $1 + 1 + 1 + 1 + 1$
" " $3a$, $1 + 3a + 9a^2 + 27a^3 + 81a^4$
Coefficients, 1 4 6 4 1
$(1 + 3a)^4 = 1 + 12a + 54a^2 + 108a^3 + 81a^4$, *Ans.*

4. The exponents decrease and increase by 2.

$(x^2 + y^2)^3 = x^6 + 3x^4y^2 + 3x^2y^4 + y^6$, *Ans.*

5. Powers of a, $a^3 + a^2 + a^1 + 1$

" " a^{-1}, $1 + a^{-1} + a^{-2} + a^{-3}$

Coefficients, 1 3 3 1

$(a + a^{-1})^3 = a^3 + 3a + 3a^{-1} + a^{-3}$, *Ans.*

6. Powers of a^2, $a^{12} + a^{10} + a^8 + a^6 + a^4 + a^2 + 1$

" " $-2a$, $1 - 2a + 4a^2 - 8a^3 + 16a^4 - 32a^5 + 64a^6$

Coefficients, 1 6 15 20 15 6 1

$(a^2 - 2a)^6 = a^{12} - 12a^{11} + 60a^{10} - 160a^9 + 240a^8 - 192a^7 + 64a^6$,

Page 288.

1. $abx^{-6} \times x^3 = abx^3$, *Ans.*
2. $a^4b^{-6}x^{-3} \times a^{-2}b^2x^{-3} = a^2b^{-4}x^{-6}$, *Ans.*
3. $x^{-m} \times x^{-n} = x^{-m-n}$, *Ans.*
4. $y^{-2} \times y^2 = y^0$ or 1, *Ans.*

1. $6a^{-5} \div 3a^{-2} = 2a^{-3}$, *Ans.*
2. $8a^{-4}bc^{-6} \div 4a^2b^4c^5 = 2a^{-6}b^{-3}c^{-11}$, *Ans.*
3. $12x^{-5} \div 4x^{-3} = 3x^{-2}$, *Ans.*
4. $(a + x)^{-n} \div (a + x)^{-m} = (a + x)^{m-n}$, *Ans.*

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Transfer by changing the sign of the exponent. (Art. 279.)

1. $\frac{x^2y}{z^3} = x^2yz^{-3}$.
2. $\frac{a^3}{b^2c^3d} = a^3b^{-2}c^{-3}d^{-1}$.
3. $\frac{3x^2}{2y^2z} = 3 \times 2^{-1}x^2y^{-2}z^{-1}$.
4. $\frac{a^2x^3cm}{ax^5cm^4} = ax^{-2}m^{-3}$.

UNITING RADICALS.

Page 289.

1. $\sqrt{48} + \sqrt{29} + \sqrt{243} = 4\sqrt{3} + 3\sqrt{3} + 9\sqrt{3}$
$= 16\sqrt{3}$, *Ans.*

2. $\sqrt{54x} - \sqrt{96x} + \sqrt{24x} = 3\sqrt{6x} - 4\sqrt{6x} + 2\sqrt{6x}$
$= \sqrt{6x}$, *Ans.*

3. $8\sqrt[3]{\frac{1}{32}b} = 8\sqrt[3]{\frac{2}{64}b} = \frac{8}{4}\sqrt[3]{2b} = 2\sqrt[3]{2b}$
$2\sqrt[3]{\frac{1}{4}b} = 2\sqrt[3]{\frac{2}{8}b} = \frac{2}{2}\sqrt[3]{2b} = \sqrt[3]{2b}$
$3\sqrt[3]{2b}$, *Ans.*

4. $x\sqrt{25x^2c} + \sqrt{36x^4c} = 5x^2\sqrt{c} + 6x^2\sqrt{c}$
$= 11x^2\sqrt{c}$, *Ans.*

5. $\sqrt{80a^4x} - \sqrt{20a^2x} = 4a^2\sqrt{5x} - 2a\sqrt{5x}$, *Ans.*

6. $3\sqrt[3]{128x^3yz} - 4x\sqrt[3]{16yz} = 12x\sqrt[3]{2yz} - 8x\sqrt[3]{2yz}$
$= 4x\sqrt[3]{2yz}$, *Ans.*

MULTIPLICATION OF RADICALS.

Page 289.

1. $(a + y)^{\frac{1}{n}} \times (b + h)^{\frac{1}{n}} = \sqrt[n]{(b + h)(a + y)}$, *Ans.*

2. $4 + 2\sqrt{2}$
$2 - \sqrt{2}$
$8 + 4\sqrt{2}$
$-4\sqrt{2} - 4$
$8 - 4$, or 4, *Ans.*

3. $(x + y)^{\frac{1}{3}} \times (x + y)^{\frac{1}{3}} = (x + y)^{\frac{2}{6}} \times (x + y)^{\frac{3}{6}}$
$= (x + y)^{\frac{5}{6}}$, *Ans.*

4. $3b\sqrt[n]{d+y} \times 4\sqrt{a} = 3b\sqrt[2n]{(d+y)^2} \times 4\sqrt[2n]{a^n}$
$= 12b\sqrt[2n]{a^n}\,(d+y)^2$, *Ans.*

DIVISION OF RADICALS.

Page 289.

1. $(a^3h)^{\frac{1}{4}} \div (ax)^{\frac{1}{4}} = \left(\frac{a^2h}{x}\right)^{\frac{1}{4}}$, *Ans.*

2. $24x\sqrt{ay} \div 6\sqrt{a} = 4x\sqrt{y}$, *Ans.*

3. $\sqrt{16a^3 - 12a^2x} \div 2a = 2a\sqrt{4a - 3x} \div 2a$
$= \sqrt{4a - 3x}$, *Ans.*

4. $(b+y)^{\frac{3}{4}} \div (b+y)^{\frac{1}{4}} = (b+y)^{\frac{1}{2}}$, *Ans.*

5. $4a\sqrt{ab} \div 2\sqrt{ac} = \frac{4a}{2}\sqrt{\frac{ab}{ac}} = 2a\sqrt{\frac{b}{c}}$
$= \frac{2a}{c}\sqrt{bc}$, *Ans.*

6. $70\sqrt[3]{9} \div 7\sqrt[3]{18} = \frac{10}{7}\sqrt[3]{\frac{9}{18}} = 10\sqrt[3]{\frac{1}{2}} = 10\sqrt[3]{\frac{4}{8}}$
$= \frac{10}{2}\sqrt[3]{4} = 5\sqrt[3]{4}$, *Ans.*

REDUCING RADICALS TO RATIONAL QUANTITIES.

Page 289.

1. $2\sqrt{a} - \sqrt{7}$, *Ans.*

See Arts. 103, 317.

2. $x - \sqrt{y}$, *Ans.*

3. $\sqrt{3}$, *Ans.*

4. $\sqrt{5} + \sqrt{x}$, *Ans.*

5. $4\sqrt{2x} + 5\sqrt{y}$, *Ans.*

6. $\sqrt{2} - \sqrt{6} + 2$, *Ans.*

This factor is obtained by trial.

$$\begin{array}{ll} \text{Multiply} & \sqrt{3} + \sqrt{2} + 1 \\ \text{By} & \sqrt{2} - \sqrt{6} + 2 \\ \hline & \sqrt{6} + \quad 2 + \sqrt{2} \\ & -\sqrt{6} \qquad -3\sqrt{2} - 2\sqrt{3} \\ & \qquad\quad 2 + 2\sqrt{2} + 2\sqrt{3} \\ \hline & \qquad\quad 4 \end{array}$$

We multiply first by $\sqrt{2}$ to rationalize the $\sqrt{2}$ in the given denominator. The product contains two radicals, $\sqrt{6}$ and $\sqrt{2}$, which may be made to disappear by multiplying by a negative quantity; we therefore try $-\sqrt{6}$, and the result, $2 - 2\sqrt{2} - 2\sqrt{3}$, has two negative radicals, the same as in the denominator; and we make these disappear by multiplying by $+2$.

RADICAL EQUATIONS.

Page 289.

1. Given $\sqrt{x} + 1 = \sqrt{11 + x}$

Squaring, $x + 2\sqrt{x} + 1 = 11 + x$

Reducing, $\sqrt{x} = 5$

$\therefore \quad x = 25$, *Ans.*

2. Given $\sqrt{x + 18} - \sqrt{5} = \sqrt{x - 7}$

Squaring, $x + 18 - 2\sqrt{5x + 90} + 5 = x - 7$

Reducing, $\sqrt{5x + 90} = 15$

Squaring, $5x + 90 = 225$

Transposing, $5x = 135$

$\therefore \quad x = 27$, *Ans.*

3. Given, $\sqrt{x^2 - 11} = 5$

Squaring, $x^2 - 11 = 25$

Transposing, $x^2 = 36$

Extracting root, $x = 6$, *Ans.*

4. Given $\dfrac{6}{\sqrt{3+x}} = \sqrt{3+x}$

Clear of fraction, $6 = 3 + x$

Transposing, $x = 3$, *Ans.*

5. Given $(13 + \sqrt{23 + y^2})^{\frac{1}{2}} = 5$

Squaring, $13 + \sqrt{23 + y^2} = 25$

Transposing, $\sqrt{23 + y^2} = 12$

Squaring, $23 + y^2 = 144$

Transposing, $y^2 = 121$

$\therefore \quad y = 11$, *Ans.*

6. Given $2\sqrt{a} = \sqrt{x + 3a}$

Squaring, $4a = x + 3a$

Transposing, $x = a$, *Ans.*

QUADRATICS.

Page 290.

1. Given $5x - \dfrac{3x - 3}{x - 3} = 2x + \dfrac{3x - 6}{2}$

Transposing and clearing of fractions,

$$6x^2 - 18x - 6x + 6 = 3x^2 - 15x + 18$$

Uniting terms, $3x^2 - 9x = 12$

Reducing, $x^2 - 3x = 4$

By Art. 334, $x = \frac{3}{2} \pm \sqrt{4 + \frac{9}{4}}$

Reducing, $x = \frac{3}{2} \pm \frac{5}{2}$

$\therefore \quad x = 4$ or -1, *Ans.*

2. Given $\frac{16}{x} - \frac{100 - 9x}{4x^2} = 3$

Clearing, $64x - 100 + 9x = 12x^2$

Transposing, $12x^2 - 73x = -100$

Dividing, $x^2 - \frac{73}{12}x = -\frac{100}{12}$

Second method, $x = \frac{73}{24} \pm \sqrt{-\frac{100}{12} + \frac{5329}{576}}$

Reducing, $x = \frac{73}{24} \pm \frac{23}{24}$

$\therefore\ x = 4$ or $2\frac{1}{12}$, *Ans.*

3. Given $\frac{x^6}{2} - \frac{x^3}{4} = -\frac{1}{32}$

Multiplying by 2, $x^6 - \frac{x^3}{2} = -\frac{1}{16}$

Second method, $x^3 = \frac{1}{4} \pm \sqrt{-\frac{1}{16} + \frac{1}{16}}$

Extracting root, $x = \sqrt[3]{\frac{1}{4}}$, *Ans.*

4. Given $\frac{\sqrt{4x} + 2}{4 + \sqrt{x}} = \frac{4 - \sqrt{x}}{\sqrt{x}}$

Clearing, $2x + 2\sqrt{x} = 16 - x$

Transposing, $2\sqrt{x} = 16 - 3x$

Squaring, $4x = 256 - 96x + 9x^2$

Or $9x^2 - 100x = -256$

Dividing, $x^2 - \frac{100}{9}x = -\frac{256}{9}$

Second method, $x = \frac{50}{9} \pm \sqrt{-\frac{256}{9} + \frac{2500}{81}}$

Reducing, $x = \frac{50}{9} \pm \frac{14}{9}$

$\therefore\ x = 7\frac{1}{9}$ or 4, *Ans.*

5. Let $x =$ greater number.

And $y =$ less number.

Then $x - y = 12$ (1)

And $x^2 + y^2 = 1424$ (2)

Squaring (1), $x^2 - 2xy + y^2 = 144$ (3)

Subtracting (3) from (2), $2xy = 1280$ (4)

Add (2) and (4), $x^2 + 2xy + y^2 = 2704$

Extract root, $x + y = 52$

Combining with (1), $x - y = 12$

We have $y = 20$;

And $x = 32$, } *Ans.*

6. Denote the numbers by x and y.

Then $x + y = 6$ (1)

And $x^3 + y^3 = 72$ (2)

Dividing (2) by (1), $x^2 - xy + y^2 = 12$ (3)

Squaring (1), $x^2 + 2xy + y^2 = 36$ (4)

Subtracting (3) from (4), $3xy = 24$

Or $xy = 8$ (5)

Subt. (5) from (3), $x^2 - 2xy + y^2 = 4$

Extract root, $x - y = 2$

Combine with (1), $x + y = 6$

By addition, $x = 4$;

By subtraction, $y = 2$, } *Ans.*

7. Denote one part by x.

Then the other part $= 56 - x$

By condition, $56x - x^2 = 640$

Changing signs, $x^2 - 56x = -640$

By 2d method, $x = 28 \pm \sqrt{-640 + 784}$

Reducing, $x = 28 \pm 12$

$\therefore\ x = 40$;

And $56 - x = 16$, } *Ans.*

8. Let $x =$ B's hourly progress.

Then $x + 3 =$ A's " "

And $\frac{150}{x} =$ B's time on the road.

And $\frac{150}{x+3} + 8\frac{1}{3} =$ A's " "

By condition, $\frac{150}{x+3} + \frac{25}{3} = \frac{150}{x}$

Reducing, $\frac{6}{x+3} + \frac{1}{3} = \frac{6}{x}$

Clearing, $18x + x^2 + 3x = 18x + 54$

Or, $x^2 + 3x = 54$

By 2d method, $x = -\frac{3}{2} \pm \sqrt{54 + \frac{9}{4}}$

Reducing, $x = -\frac{3}{2} \pm \frac{15}{2}$

$\therefore\ x = 6$ miles, B's rate;

And $x + 3 = 9$ " A's " } *Ans.*

9. Let x = greater number,
And y = less number.
Then $x - y = 6$ (1)
And $2y^2 + 47 = x^2$ (2)
From (1), $x^2 = 36 + 12y + y^2$
Substituting, $2y^2 + 47 = 36 + 12y + y^2$
Transposing, $y^2 - 12y = -11$
By 2d method, $y = 6 \pm \sqrt{-11 + 36}$
Reducing, $y = 11$, less,
From (1), $x = 17$, greater, } *Ans.*

10. Denote the length and breadth by x and y.
Then $x + y = 42$ (1)
And $xy = 432$ (2)
Squaring (1), $x^2 + 2xy + y^2 = 1764$
Multiplying (2) by 4, $4xy = 1728$
Subtracting, $x^2 - 2xy + y^2 = 36$
Extract root, $x - y = 6$
Combining with (1), $x + y = 42$
By addition, $x = 24$ ft., len.;
By subtraction, $y = 18$ ft., bre., } *Ans.*

11. Let x = A's age;
And y = B's age.
Then $xy = 120$, or $x = \frac{120}{y}$ (1)
And $(x - 3)(y + 2) = 120$
Or $xy + 2x - 3y - 6 = 120$ (2)
By substi., $120 + \frac{240}{y} - 3y = 126$
Reducing, $y^2 + 2y = 80$
By 2d method, $y = -1 \pm \sqrt{80 + 1}$
Reducing, $y = 8$ yrs., B's age;
From (1), $x = 15$ yrs., A's age, } *Ans.*

12. Given $\sqrt{x^5} + \sqrt{x^3} = 6\sqrt{x}$
Squaring, $x^5 + 2x^4 + x^3 = 36x$
Dividing by x, $x^4 + 2x^3 + x^2 = 36$
Extracting root, $x^2 + x = 6$
By 2d Method, $x = -\frac{1}{2} \pm \sqrt{6 + \frac{1}{4}}$
Reducing, $x = -\frac{1}{2} \pm \frac{5}{2}$
$\therefore\ x = 2$, *Ans.*

13. Given $x + \sqrt{x+6} = 2 + 3\sqrt{x+6}$
Transposing, $x - 2 = 2\sqrt{x+6}$
Squaring, $x^2 - 4x + 4 = 4x + 24$
Uniting terms, $x^2 - 8x = 20$
By 2d method, $x = 4 \pm \sqrt{20 + 16}$
Or $x = 4 \pm 6$
$\therefore\ x = 10$, or -2, *Ans.*

14. Let x = No. lbs. pepper for £10.
Then $x + 60$ = " ginger for £20.
And $\frac{10}{x}$ = Price of pepper per pound.
And $\frac{20}{x+60}$ = " ginger "
By conditions, $80 \times \frac{10}{x} + 100 \times \frac{20}{x+60} = 65$
Reducing, $\frac{160}{x} + \frac{400}{x+60} = 13$
Clearing, $160x + 9600 + 400x = 13x^2 + 780x$
Uniting and dividing, $x^2 + \frac{220}{13}x = \frac{9600}{13}$
By 2d method, $x = -\frac{110}{13} \pm \sqrt{\frac{9600}{13} + \frac{12100}{13^2}}$
Reducing, $x = -\frac{110}{13} \pm \frac{370}{13}$
$\therefore\ x = 20$

And price of pepper per lb. $\frac{10}{x} = \frac{10}{20} = \frac{1}{2}$£, or 10s.;
" " ginger " $\frac{20}{x+60} = \frac{20}{80} = \frac{1}{4}$£, or 5s.
} *Ans*

COLLEGE EXAMINATION PROBLEMS.

Page 291, Art. 542.

1. $\left(15x^2 - \frac{3ab}{5c}\right) \div \left(x - \frac{a-b}{c}\right)$;

$$\frac{75cx^2 - 3ab}{5c} \times \frac{c}{cx - a + b} = \frac{75cx^2 - 3ab}{5(cx - a + b)}, \quad \textit{Ans.}$$

2. $(a^4 - b^4) \div (a - b) = a^3 + a^2b + ab^2 + b^3$, *Ans.*

See Art. 129, Prin. 1.

3. Given $x + \frac{3x-5}{2} = 12 - \frac{2x-4}{3}$

Clearing of fractions, $6x + 9x - 15 = 72 - 4x + 8$

Uniting terms, $19x = 95$

$\therefore \quad x = 5$, *Ans.*

4. Multiply $3\sqrt{45} - 7\sqrt{5} = 2\sqrt{5}$

By $\sqrt{1\tfrac{4}{5}} + 2\sqrt{9\tfrac{4}{5}} = 17\sqrt{\tfrac{1}{5}}$

And $17\sqrt{\tfrac{1}{5}} \times 2\sqrt{5} = 34$, *Ans.*

5. $a^{\frac{1}{2}}b^{\frac{1}{3}} \div a^{\frac{1}{3}}b^{\frac{1}{4}} = a^{\frac{1}{6}}b^{\frac{1}{12}}$, *Ans.*

Subtract the exponent of the divisor from that of the dividend.

6. $xy^{-1} \div x^{\frac{2}{3}}y^{-\frac{1}{2}} = x^{\frac{1}{3}}y^{-\frac{1}{2}}$, *Ans.*

7. Given $3x^2 + 2x - 9 = 76$

Reducing, $x^2 + \frac{2}{3}x = \frac{85}{3}$

By 2d method, $x = -\frac{1}{3} \pm \sqrt{\frac{85}{3} + \frac{1}{9}}$

Reducing, $x = -\frac{1}{3} \pm \frac{16}{3}$

$\therefore \quad x = 5$, or $-5\frac{2}{3}$, *Ans.*

8. Given $\frac{1}{2}x^2 - \frac{1}{3}x + 7\frac{3}{8} = 8$

Transposing, etc., $x^2 - \frac{2}{3}x = \frac{5}{4}$

By 2d method, $x = \frac{1}{3} \pm \sqrt{\frac{5}{4} + \frac{1}{9}}$

Reducing, $x = \frac{1}{3} \pm \frac{7}{6}$

$\therefore \quad x = 1\frac{1}{2}$, or $-\frac{5}{6}$, *Ans.*

9. Denote the numbers by x and y.

Then $x : y :: x + y : 42$ (1)

And $x : y :: x - y : 6$ (2)

From (1), (Art. 378.) $42x = xy + y^2$

From (2), $6x = xy - y^2$ (3)

Subtracting, $36x = 2y^2$

Or $x = \frac{y^2}{18}$ (4)

Substituting in (3), $\frac{y^2}{3} = \frac{y^2}{18} - y^2$

Dividing by y^2, $\frac{1}{3} = \frac{y}{18} - 1$

Reducing, $y = 24$, less;
From (4), $x = 32$, gr., } *Ans.*

10. $S = \frac{a}{1 - r} = \frac{1}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = 1\frac{1}{2}$, *Ans.* (Art. 435.)

11. $(a + b)^{15} = a^{15} + 15a^{14}b + 105a^{13}b^2$, etc., *Ans.*

12. $(1 + x^2)^{-\frac{1}{5}} = 1 - \frac{x^2}{5} + \frac{3x^4}{25} - \frac{11x^6}{125}$, *Ans.* (Art. 270.)

$n = -\frac{1}{5}$;

$n \times \frac{n-1}{2} = -\frac{1}{5} \times \frac{-\frac{1}{5} - 1}{2} = -\frac{1}{5} \times -\frac{3}{5} = \frac{3}{25}$;

$n \times \frac{n-1}{2} \times \frac{n-2}{3} = \frac{3}{25} \times \frac{-\frac{1}{5} - 2}{3}$

$= \frac{3}{25} \times -\frac{11}{15} = -\frac{11}{125}$.

Page 291, Art. 543.

1. $\left(\frac{x+2y}{x+y}+\frac{x}{y}\right) \div \left(\frac{x+2y}{y}-\frac{x}{x+y}\right)$;

$\frac{xy+2y^2+x^2+xy}{(x+y)\,y} \div \frac{x^2+3xy+2y^2-xy}{(x+y)\,y}$;

$\frac{2xy+2y^2+x^2}{(x+y)\,y} \times \frac{(x+y)\,y}{2xy+2y^2+x^2} = 1$, *Ans.*

2. $a^{\frac{1}{2}}a^{\frac{1}{4}}a^{\frac{1}{4}}a^{-\frac{1}{2}} = a^{\frac{6}{8}}a^{\frac{4}{8}}a^{\frac{3}{8}}a^{-\frac{1}{8}} = a$, *Ans.*

3. Given $x+\sqrt{a^2+x^2} = \frac{2a^2}{\sqrt{a^2+x^2}}$

Clearing, $x\sqrt{a^2+x^2}+a^2+x^2 = 2a^2$

Transposing, $x\sqrt{a^2+x^2} = a^2-x^2$

Squaring, $a^2x^2+x^4 = a^4-2a^2x^2+x^4$

Uniting terms, $3a^2x^2 = a^4$

Or $x^2 = \frac{a^2}{3}$

$\therefore\ x = \pm a\sqrt{\frac{1}{3}}$, or $\pm\frac{a}{3}\sqrt{3}$, *Ans.*

4. Given $\frac{90}{x}-\frac{90}{x+1}-\frac{27}{x+2} = 0$

Then $10x^2+30x+20-10x^2-20x-3x^2-3x = 0$

Reducing, $3x^2-7x = 20$

Dividing by 3, $x^2-\frac{7}{3}x = \frac{20}{3}$

By 2d method, $x = \frac{7}{6}\pm\sqrt{\frac{20}{3}+\frac{49}{36}}$

Extracting root, $x = \frac{7}{6}\pm\frac{17}{6}$

$\therefore\ x = 4$, or $-1\frac{2}{3}$, *Ans.*

5. Given $\sqrt{x-1} = x-1$

Squaring, $x-1 = x^2-2x+1$

Transposing, $x^2-3x = -2$

By 2d method, $x = \frac{3}{2}\pm\sqrt{-2+\frac{9}{4}}$

Reducing, $x = \frac{3}{2}\pm\frac{1}{2}$

$\therefore\ x = 2$, or 1, *Ans.*

6. $s = \frac{a}{1-r} = \frac{4}{1-\frac{3}{4}} = \frac{4}{\frac{1}{4}} = 4\times 4 = 5\frac{1}{3}$, *Ans.* See Art. 435.

7. Given $x + \sqrt{x} : x - \sqrt{x} :: 3\sqrt{x} + 6 : 2\sqrt{x}$

By Theorem 7, $2x : x - \sqrt{x} :: 5\sqrt{x} + 6 : 2\sqrt{x}$

By Theorem 1, $4x\sqrt{x} = 5x\sqrt{x} + x - 6\sqrt{x}$

Reducing, $4x = 5x + \sqrt{x} - 6$

Or, $\sqrt{x} = x - 6 \quad (1)$

Squaring, $x = x^2 - 12x + 36 \quad (2)$

Transposing, $x^2 - 13x = -36$

By 2d method, $x = 13/2 \pm \sqrt{-36 + 169/4}$

Or, $x = 13/2 \pm 5/2 = 9$ or 4.

It may be observed that this is a peculiar case. Both values of x satisfy equation (2); while equation (1) is only satisfied with the second value 4. The first value 9, only reaches backward to equation (2); while the second value 4, verifies the given proportion. Hence, $x = 4$, *Ans.*

8. $(a^2+x)^{12} = a^{24} + 12a^{22}x + 66a^{20}x^2 + 220a^{18}x^3 + \text{etc.}$, *Ans.*

See Arts. 270, 271.

$$n = 12; \quad n \times \frac{n-1}{2} = \frac{12 \times 11}{2} = 66;$$

$$n \times \frac{n-1}{2} \times \frac{n-2}{3} = 66 \times \frac{10}{3} = 220.$$

9. $$(x^2 - y^2)^{-\frac{1}{4}} = \frac{1}{x^{\frac{1}{2}}}\left(1 - \frac{y^2}{x^2}\right)^{-\frac{1}{4}}$$

$$= \frac{1}{x^{\frac{1}{2}}}\left(1 + \frac{y^2}{4x^2} + \frac{5y^4}{32x^4} + \frac{15y^6}{128x^6}, \text{ etc.}\right)$$

$$= \frac{1}{x^{\frac{1}{2}}} + \frac{y^2}{4x^{\frac{5}{2}}} + \frac{5y^4}{32x^{\frac{9}{2}}} + \frac{15y^6}{128x^{\frac{13}{2}}}, \text{ etc., } \textit{Ans.}$$

$n = -1/4$ Arts. 270, 271.

$$n \times \frac{n-1}{2} = -\frac{1}{4} \times \frac{-\frac{1}{4} - 1}{2}$$

$$= -\frac{1}{4} \times -\frac{5}{8} = \frac{5}{32}$$

$$n \times \frac{n-1}{2} \times \frac{n-2}{3} \overset{\bullet}{=} \frac{5}{32} \times \frac{-\frac{1}{4}-2}{3} = \frac{5}{32} \times -\frac{3}{4}$$
$$= -\frac{15}{128}.$$

Page 291, Art. 544.

1. $243a^{10}b^5+1=(3a^2b+1)(81a^8b^4-27a^6b^3+9a^4b^2-3a^2b+1)$

$81a^8b^4-1=(9a^4b^2+1)(3a^2b+1)(3a^2b-1)$. (Art. 129.)

Hence, *g. c. d.* $= 3a^2b + 1$, *Ans.*

Again, the *l. c. m.* of these factors

$$= (9a^4b^2 + 1)(3a^2b + 1)(3a^2b - 1)(81a^8b^4 - 27a^6b^3 + 9a^4b^2 - 3a^2b + 1)$$
$$= (81a^8b^4-1)(81a^8b^4-27a^6b^3+9a^4b^2-3a^2b+1)$$
$$= 6561a^{16}b^8 - 2187a^{14}b^7 + 729a^{12}b^6 - 243a^{10}b^5 + 27a^6b^3 - 9a^4b^2 + 3a^2b - 1, \text{ Ans.}$$

2. Divide $\frac{6\sqrt{b}}{25\sqrt[5]{a^3}}$ by $\frac{20c\sqrt[4]{b^3}}{21ab\sqrt[3]{a^2}}$

$$\frac{6b^{\frac{1}{2}}}{25a^{\frac{3}{5}}} \div \frac{20cb^{\frac{3}{4}}}{21a^{\frac{5}{3}}b}$$
$$\frac{6 \times 21a^{\frac{5}{3}}b^{\frac{3}{2}}}{25 \times 20a^{\frac{3}{5}}b^{\frac{3}{4}}c} = \frac{63}{250}a^{\frac{16}{15}}b^{\frac{3}{4}}c^{-1}, \text{ Ans.}$$

3. Given $2x - y = 21$

Or $y = 2x - 21$ (1)

And $2x^2 + y^2 = 153$ (2)

Squaring (1), $y^2 = 4x^2 - 84x + 441$

Substituting in (2),

$$2x^2 + 4x^2 - 84x + 441 = 153$$

Reducing, $x^2 - 14x = -48$

By 2d method, $x = 7 \pm \sqrt{-48 + 49}$

$\therefore x = 8$, or 6;

And from (1), $y = -5$, or -9. } *Ans.*

Page 292, Art. 544.

4. Let x = No. of yards.

Then $\frac{90}{x}$ = price per yard.

By conditions, $\frac{90}{x} = \frac{90}{x+2} + \frac{1}{2}$

Clearing, $180x + 360 = 180x + x^2 + 2x$

Or $x^2 + 2x = 360$

By 2d method, $x = -1 \pm \sqrt{361}$

$= -1 \pm 19$

$\therefore\ x = 18$ yards;

And $\frac{90}{x} = \$5$, price, } *Ans.*

5. $(a-b)^{12} = a^{12} - 12a^{11}b + 66a^{10}b^2 - 220a^9b^3 + 495a^8b^4$
$- 792a^7b^5 + 924a^6b^6 - 792a^5b^7 + 495a^4b^8$
$- 220a^3b^9 + 66a^2b^{10} - 12ab^{11} + b^{12}$, *Ans.*

6. $4a^2 - 9y^2 = (2a + 3y)(2a - 3y)$, *Ans.* (Art. 103.)

7. $3\sqrt{\frac{a}{3}} \times 2\sqrt{\frac{a}{6}} = 6\sqrt{\frac{a^2}{18}} = 6\sqrt{\frac{2a^2}{36}} = \frac{6a}{6}\sqrt{2}$
$= a\sqrt{2}$, *Ans.*

8. Given $x + 2y = 7$ (1)

And $2x + 3y = 12$ (2)

Mult. (1) by 2, $2x + 4y = 14$ (3)

Subt. (2) from (3), $y = 2$;

Substituting in (1), $x = 3$, } *Ans.*

9. $a\sqrt{48a^3d} = a\sqrt{16a^2 \times 3ad} = 4a^2\sqrt{3ad}$;

$\sqrt{\frac{45}{49}} = \sqrt{\frac{9}{49} \times 5} = \frac{3}{7}\sqrt{5}$, } *Ans.*

10. Given $\frac{x-5}{3} + \frac{x}{2} = 12 - \frac{x-10}{3}$

Clearing of frac., $2x - 10 + 3x = 72 - 2x + 20$

Uniting terms, $7x = 102$

$\therefore\ x = 14\frac{4}{7}$, *Ans.*

Page 292, Art. 545.

1. From $3x + \frac{x}{2b}$ subtract $x - \frac{x-a}{c}$

Ans. $2x + \frac{x}{2b} + \frac{x-a}{c}$, or $2x + \frac{cx + 2b(x-a)}{2bc}$

2. $\frac{1-x^2}{1+y} \times \frac{1-y^2}{x+x^2} \times \left(1 + \frac{x}{1-x}\right)$, or

$\frac{(1+x)(1-x)(1+y)(1-y)}{(1+y)x(1+x)(1-x)} = \frac{1-y}{x}$, *Ans.*

3. $a^4 - 4a^3b + 8ab^3 + 4b^4 \;(\; a^2 - 2ab - 2b^2$, *Ans.*

a^4

$2a^2 - 2ab \;)\; -4a^3b$

$-4a^3b + 4a^2b^2$

$2a^2 - 4ab - 2b^2 \;)\; -4a^2b^2 + 8ab^3 + 4b^4$

$-4a^2b^2 + 8ab^3 + 4b^4$

4. From $2\sqrt[3]{320} = 8\sqrt[3]{5}$

Take $3\sqrt[3]{40} = 6\sqrt[3]{5}$

$2\sqrt[3]{5}$, *Ans.*

5. $a^{-\frac{1}{2}}b^{\frac{1}{2}} \div a^{\frac{1}{3}}b^{-\frac{1}{4}} = a^{-\frac{5}{6}}b^{\frac{7}{12}}$, *Ans.*

6. Given $x^4 + 4x^2 = 12$

By 2d method, $x^2 = -2 \pm \sqrt{12+4}$

Reducing, $x^2 = -2 \pm 4 = 2$, or -6

Extracting root, $x = \pm\sqrt{2}$, or $\pm\sqrt{-6}$, *Ans.*

7. Given $x^2 - x\sqrt{3} = x - \frac{1}{2}\sqrt{3}$

Transposing, $x^2 - (1 + \sqrt{3})x = -\frac{\sqrt{3}}{2}$

By 2d method,

$$x = \frac{1+\sqrt{3}}{2} \pm \sqrt{-\frac{\sqrt{3}}{2} + \frac{1 + 2\sqrt{3} + 3}{4}}$$

Reducing, $x = \frac{1+\sqrt{3}}{2} \pm 1$

$\therefore \quad x = \frac{3+\sqrt{3}}{2}$, or $\frac{\sqrt{3}-1}{2}$, *Ans.*

8. Denote the numbers by x and y.

Then $x + y = 2a$ (1)

And $x^2 + y^2 = 2b$ (2)

Squaring (1),

$x^2 + 2xy + y^2 = 4a^2$ (3)

Subt. (2) from (3), $2xy = 4a^2 - 2b$ (4)

" (4) " (2),

$x^2 - 2xy + y^2 = 4b - 4a^2$

Extracting root, $x - y = \pm 2\sqrt{b - a^2}$ (5)

Combining (1) and (5), $x = a \pm \sqrt{b - a^2}$;

And $y = a \mp \sqrt{b - a^2}$, } *Ans.*

9. Denote the numbers by x and y.

Then $x - y : x + y :: 2 : 3$ (1)

And $x - y : xy :: 2 : 5$ (2)

From (1), (Th. 7) $2x : x + y :: 5 : 3$

Theorem 1, $6x = 5x + 5y$

Or $x = 5y$ (3)

From (2), (Th. 1) $5x - 5y = 2xy$

Substituting value of x, $10y = 10y^2$

Reducing, $y = 2$;

From (3), $x = 10$, } *Ans.*

10. $r = \left(\frac{l}{a}\right)^{\frac{1}{n+1}} = (\frac{162}{2})^{\frac{1}{4}} = \sqrt[4]{81} = 3.$

See Art. 407.

Hence the series 2, 6, 18, 54, 162, *Ans.*

11. $\frac{m}{\sqrt{x^2+a^2}} = m(x^2+a^2)^{-\frac{1}{2}} = \frac{m}{x}\left(1+\frac{a^2}{x^2}\right)^{-\frac{1}{2}}$;

$\frac{m}{x}\left(1+\frac{a^2}{x^2}\right)^{-\frac{1}{2}} = \frac{m}{x}\left(1-\frac{a^2}{2x^2}+\frac{3a^4}{8x^4}-\frac{15a^6}{48x^6}, \text{ etc.}\right)$, or

$m\left(\frac{1}{x}-\frac{a^2}{2x^3}+\frac{3a^4}{8x^5}-\frac{15a^6}{48x^7}, \text{ etc.}\right)$, *Ans.*

See Art. 270. $n = -\frac{1}{2}$;

$n \times \frac{n-1}{2} = -\frac{1}{2} \times \frac{-\frac{1}{2}-1}{2} = -\frac{1}{2} \times -\frac{3}{4} = \frac{3}{8}$;

$n \times \frac{n-1}{2} \times \frac{n-2}{3} = \frac{3}{8} \times \frac{-\frac{1}{2}-2}{3}$

$= \frac{3}{8} \times -\frac{5}{6} = -\frac{15}{48}$.

Page 292, Art. 546.

1. $\frac{12x^4-192}{3x-6} = \frac{12(x^4-16)}{3(x-2)} = 4x^3+8x^2+16x+32$, *Ans.*

2. Divide $x^2+\frac{x^3}{a-b}$ by $\frac{ab}{a-b}-m$

Reducing, $\frac{x^2(a-b)+x^3}{a-b} \div \frac{ab-m(a-b)}{a-b}$

Dividing, $\frac{x^2(a-b)+x^3}{ab-m(a-b)}$, *Ans.*

3. Given $21+\frac{3x-11}{16} = \frac{5x-5}{8}+\frac{97-7x}{2}$

Clear. of frac., $336+3x-11 = 10x-10+776-56x$

Uniting terms, $49x = 441$, $x = 9$, *Ans.*

4. $a^{\frac{1}{2}}a^{\frac{2}{3}}a^{\frac{3}{4}}a^{-\frac{4}{5}} = a^{\frac{67}{60}}$, *Ans.*

$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}-\frac{4}{5} = \frac{30+40+45-48}{60} = \frac{67}{60}$, exponent.

5. Given $\frac{15}{x}-\frac{72-6x}{2x^2} = 2$

Clearing of frac., $30x-72+6x = 4x^2$

Reducing, $x^2-9x = -18$

By 2d method, $x = \frac{9}{2} \pm \sqrt{-18+\frac{81}{4}}$

$\therefore\ x = \frac{9}{2} \pm \frac{3}{2} = 6$, or 3, *Ans.*

6. See Art. 394. $d = \frac{l - a}{m + 1} = \frac{50 - 1}{6 + 1} = 7.$

Hence the series,

1, 8, 15, 22, 29, 36, 43, 50, *Ans.*

Page 293, Art. 546.

7. RULE.—Divide the product of the natural numbers from m down to $m - n + 1$ inclusive, by the product of the natural numbers from 1 to n inclusive.

Formula, $C = \frac{m \,..\, (m-n+1)}{1 \cdot 2 \cdot 3 \,....\, n} = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$, *Ans.*

NOTE.—m denotes the whole number of letters, and n the number of letters taken in a set.

8. $(a - b)^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}\left(1 - \frac{b}{a}\right)^{-\frac{1}{2}}$

$$= \frac{1}{a^{\frac{1}{2}}}\left(1 + \frac{b}{4a} + \frac{5b^2}{32a^2} + \frac{15b^3}{128a^3}, \text{ etc.}\right);$$

$$(a - b)^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} + \frac{b}{4a^{\frac{3}{2}}} + \frac{5b^2}{32a^{\frac{5}{2}}} + \frac{15b^3}{128a^{\frac{7}{2}}}, \text{ etc., } \textit{Ans.}$$

9. Denote the smaller by x.

Then $x : 150 - x :: 7 : 8$.

By Theorem 1, $8x = 1050 - 7x$

Or $15x = 1050$

$\therefore\ x = 70$, smaller;
$150 - x = 80$, greater, } *Ans.*

10. Given $5x + 2y = 29$

And $\underline{-x + 2y = -1}$

Subtracting, $6x = 30$

$\therefore\ x = 5$;
Substituting, $y = 2$, } *Ans.*

Page 293, Art. 547.

1. Given $\frac{2}{2+y} - \frac{1}{y-2} + \frac{3}{5} = 0$

$10y - 20 - 10 - 5y + 3y^2 - 12 = 0$

Uniting terms, $3y^2 + 5y = 42$

Dividing, $y^2 + \frac{5}{3}y = 14$

By 2d method, $y = -\frac{5}{6} \pm \sqrt{14 + \frac{25}{36}}$

Reducing, $y = -\frac{5}{6} \pm \frac{23}{6}$

$\therefore\ y = 3$, or $-4\frac{2}{3}$, *Ans.*

2. $a^0 \div a^1 = a^{-1}$; $a^{-1} \div a^0 = a^{-1}$. See Art. 396.

Hence, a, a^0, a^{-1} are part of a geometrical series of which the ratio $r = a^{-1}$, *Ans.*

3. Let $2x =$ first number,

And $4y =$ second number.

Then $x + 3y = 11$ (1)

And $x + 3y = 6x - 4y$ (2)

From (2), $x = \frac{7y}{5}$ (3)

Substituting in (1), $\frac{7y}{5} + 3y = 11$

Or $22y = 55$

$2y = 5$

$y = \frac{5}{2}$

From (3), $x = \frac{7}{5} \times \frac{5}{2}$

$\therefore\ 4y = 10$, second;

And $2x = 7$, first, } *Ans.*

4. Powers of $2a$,

$256a^8 + 128a^7 + 64a^6 \ldots 4a^2 + 2a + 1$

Powers of $-\frac{b}{3}$,

$1 - \frac{b}{3} + \frac{b^2}{9} \ldots \frac{b^6}{729} - \frac{b^7}{2187} + \frac{b^8}{6561}$

Coefficients,

1 8 28 .. 28 8 1

$$\left(2a - \frac{b}{3}\right)^8 = 256a^8 - \frac{1024a^7b}{3} + \frac{1792a^6b^2}{9} \ldots \frac{112a^2b^6}{729} - \frac{16ab^7}{2187} + \frac{b^8}{6561},\ \textit{Ans.}$$

5. $a^2 - b^2 = (a + b)(a - b)$;
$a^2 = 2ab + b^2 = (a - b)^2$.
Hence the *g. c. d.* is $a - b$, *Ans.*

6. $a^2 - x^2 = (a + x)(a - x)$;
Hence *l. c. m.* is $4a^2 - 4x^2$, *Ans.*

7. $\frac{13a - 29b}{5(a - b)} - \frac{7b - 21a}{5(a - b)} + \frac{9b - 11a}{5(a - b)^2}$, or
$\frac{13a^2 - 42ab + 29b^2 - 28ab + 21a^2 + 7b^2 + 9b - 11a}{5(a - b)^2}$, or
$\frac{34a^2 - 63ab + 36b^2 + 9b - 11a}{5(a - b)^2}$, *Ans.*

8. Given $\sqrt{x + a} = \sqrt{x} + a$
Squaring $x + a = x + 2a\sqrt{x} + a^2$
Reducing, $2\sqrt{x} = 1 - a$
Squaring, $4x = 1 - 2a + a^2$
$\therefore\ x = \frac{1 - 2a + a^2}{4}$, *Ans.*

Page 293, Art. 548.

1. $\frac{a^3 + a^2x - ax^2 - x^3}{a^2 - x^2} = \frac{(a+x)^2(a-x)}{(a+x)(a-x)} = a + x$, *Ans.*

2. $a^{-6}b^2 \times \frac{\sqrt{b}}{\sqrt[3]{a^2}} = \frac{b^2\sqrt{b}}{a^6\sqrt[3]{a^2}}$, or $\frac{b^{\frac{5}{2}}}{a^{\frac{20}{3}}}$, or $a^{-\frac{20}{3}}b^{\frac{5}{2}}$;
$a^{-6}b^2 \div \frac{\sqrt{b}}{\sqrt[3]{a^2}} = \frac{a^{-6}b^2\sqrt[3]{a^2}}{\sqrt{b}}$, or $\frac{b^{\frac{3}{2}}}{a^{\frac{16}{3}}}$, or $a^{-\frac{16}{3}}b^{\frac{3}{2}}$, } *Ans.*

3. Given $\frac{7x - 6}{35} - \frac{x - 5}{6x - 101} = \frac{x}{5}$
Multiplying by 35, $7x - 6 - \frac{35x - 175}{6x - 101} = 7x$
Transposing, etc., $36x - 606 + 35x - 175 = 0$
Uniting terms, $71x = 781$
$\therefore\ x = 11$, *Ans.*

4. Given $\frac{7x+9}{4} - \left(x - \frac{2x-1}{9}\right) = 7$

Clearing of fractions, $63x + 81 - 28x - 4 = 252$

Reducing, $35x = 175$

$\therefore \quad x = 5$, *Ans.*

5. Given $\frac{x^2}{2} - \frac{x}{3} + 7\frac{3}{8} = 8$

Trans. and mult. by 2, $x^2 - \frac{2}{3}x = \frac{5}{4}$

By 2d method, $x = \frac{1}{3} \pm \sqrt{\frac{5}{4} + \frac{1}{9}}$

Reducing, $x = \frac{1}{3} \pm \frac{7}{6}$

$\therefore \quad x = 1\frac{1}{2}$, or $-\frac{5}{6}$, *Ans.*

6. Denote the numbers by x, y, and z.

Then $xy = 15$

Or $x = \frac{15}{y}$ (1)

And $xz = 21$

Or $x = \frac{21}{z}$ (2)

And $y^2 + z^2 = 74$ (3)

Equating (1) and (2), $\frac{15}{y} = \frac{21}{z}$

$y = \frac{15z}{21}$

$y^2 = \frac{225z^2}{441}$

Substituting in (3), $\frac{225z^2}{441} + z^2 = 74$

Or $666z^2 = 74 \times 441$

$\therefore \quad z^2 = 49$

Extracting root, $z = 7$, 3d;
Substituting in (2), $x = 3$, 1st;
Substituting in (1), $y = 5$, 2d, } *Ans.*

Page 294, Art. 548.

7. $l = a + (n-1)\,d = 1 + (n-1) \times 1 = n$;
See Art. 388.
$S = \frac{a+l}{2} \times n = \frac{1+n}{2} \times n = \frac{n+n^2}{2}$, *Ans.*
See Art. 389.

8. $(a^3 - b^3)^{-\frac{1}{3}} = \frac{1}{a}\left(1 - \frac{b^3}{a^3}\right)^{-\frac{1}{3}}$
$= \frac{1}{a}\left(1 + \frac{b^3}{3a^3} + \frac{2b^6}{9a^6} + \frac{14b^9}{81a^9} + \frac{35b^{12}}{243a^{12}}, \text{ etc.}\right)$
$= 1 + \frac{b^3}{3a^4} + \frac{2b^6}{9a^7} + \frac{14b^9}{81a^{10}} + \frac{35b^{12}}{243a^{13}}, \text{ etc.}, \textit{Ans.}$
$n = -\frac{1}{3}$;
$n \times \frac{n-1}{2} = -\frac{1}{3} \times \frac{-\frac{1}{3}-1}{2} = -\frac{1}{3} \times -\frac{2}{3} = \frac{2}{9}$;
$n \times \frac{n-1}{2} \times \frac{n-2}{3} = \frac{2}{9} \times \frac{-\frac{1}{3}-2}{3} = \frac{2}{9} \times -\frac{7}{9} = -\frac{14}{81}$;
$n \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4}$
$= -\frac{14}{81} \times \frac{-\frac{1}{3}-3}{4} = -\frac{14}{81} \times -\frac{5}{6} = \frac{35}{243}$.

9. $\sqrt{300} + \sqrt{75} = 10\sqrt{3} + 5\sqrt{3} = 15\sqrt{3}$, *Ans.*

10. Given $\frac{x}{7} + \frac{21}{x+5} = \frac{23}{7}$
Clearing of frac., $x^2 + 5x + 147 = 23x + 115$
Transposing, $x^2 - 18x = -32$
By 2d method, $x = 9 \pm \sqrt{-32+81}$
Or $x = 9 \pm 7$
$\therefore\ x = 16$, or 2, *Ans.*

Page 294, Art. 549.

1. $\sqrt{18a^5b^3} \pm \sqrt{50a^3b^3} = 3a^2b\sqrt{2ab} \pm 5ab\sqrt{2ab}$, *Ans.*

2. Multiply $2\sqrt{3} - \sqrt{-5}$

By $4\sqrt{3} - 2\sqrt{-5}$

$24 - 4\sqrt{-15}$

$-4\sqrt{-15} + 2 \times -5$

(Arts. 312, 514.) $14 - 8\sqrt{-15}$, *Ans.*

3. Given $\frac{x-1}{7} + \frac{23-x}{5} = 7 - \frac{4+x}{4}$

Clearing, $20x - 20 + 644 - 28x = 980 - 140 - 35x$

Uniting terms, $27x = 216$

$\therefore \quad x = 8$, *Ans.*

4. Given $\frac{x-3}{x-2} - \frac{x-4}{x-1} = \frac{7}{20}$

Clearing of fractions,

$20(x^2 - 4x + 3 - x^2 + 6x - 8) = (x^2 - 3x + 2)\,7$

Or $40x - 100 = 7x^2 - 21x + 14$

Trans. and divid., $x^2 - \frac{61}{7}x = -\frac{114}{7}$

By 2d method, $x = \frac{61}{14} \pm \sqrt{-\frac{114}{7} + \frac{3721}{196}}$

Reducing, $x = \frac{61}{14} \pm \frac{23}{14}$

$\therefore \quad x = 6$, or $2\frac{5}{7}$, *Ans.*

5. See Art. 392. $n = \frac{2s}{a+l} = \frac{2 \times 198}{2+42} = 9$;

See Art. 391. $d = \frac{l-a}{n-1} = \frac{42-2}{9-1} = 5$, } *Ans.*

6. $(a^3 - b^3)^{\frac{1}{3}} = a\left(1 - \frac{b^3}{a^3}\right)^{\frac{1}{3}} = a\left(1 - \frac{b^3}{3a^3} - \frac{b^6}{9a^6} - \frac{5b^9}{81a^9}, \text{ etc.}\right)$;

$n = \frac{1}{3}$; $n \times \frac{n-1}{2} = \frac{1}{3} \times \frac{\frac{1}{3}-1}{2} = -\frac{1}{9}$;

$n \times \frac{n-1}{2} \times \frac{n-2}{3} = -\frac{1}{9} \times \frac{\frac{1}{3}-2}{3} = -\frac{1}{9} \times -\frac{5}{9} = \frac{5}{81}$;

$(a^3 - b^3)^{\frac{1}{3}} = a - \frac{b^3}{3a^2} - \frac{b^6}{9a^5} - \frac{5b^9}{81a^8}$, etc., *Ans.*

7. $(a^3 - 2a^2b + ab^2)^{\frac{1}{2}} = \sqrt{(a^2 - 2ab + b^2)\,a}$
$= (a - b)\sqrt{a}$, *Ans.*

8. Let x, y and z be the times required by A, B, and C.

Then $\frac{1}{x} + \frac{1}{y} = \frac{4}{15} = \frac{8}{30}$ (1)

Part A and B do in 1 day;

$\frac{1}{y} + \frac{1}{-} = \frac{7}{30}$ (2)

Part B and C do in 1 day;

$\frac{1}{x} + \frac{1}{z} = \frac{1}{6} = \frac{5}{30}$ (3)

Part A and C do in 1 day;

Adding and reducing, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{10}{30}$ (4)

Part A, B and C do in 1 day;

Subtracting (2) from (4), $\frac{1}{x} = \frac{1}{10}$

Part A does in 1 day;

Subtracting (3) from (4), $\frac{1}{y} = \frac{1}{6}$

Part B does in 1 day;

Subtracting (1) from (4), $\frac{1}{z} = \frac{1}{15}$

Part C does in 1 day.

Clearing of fractions, $x = 10$ days, A's time;
$y = 6$ days, B's time;
$z = 15$ days, C's time. *Ans.*

If all united do $\frac{1}{3}$ in 1 day, it will take 3 days to finish.

9. 3 = First number;
5 = Second number; *Ans.*
7 = Third number,

See solution of Ex. 6, under Art. 548.

Page 294, Art. 550.

1. Given
$$\frac{1}{x}+\frac{1}{y}=2 \qquad (1)$$
$$\frac{1}{x}+\frac{1}{z}=3 \qquad (2)$$
$$\frac{1}{y}+\frac{1}{z}=3 \qquad (3)$$

Add and div. by 2, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=4$ (4)

Subtracting (3) from (4), $\frac{1}{x}=1$, or $x=1$;
" (2) " $\frac{1}{y}=1$, or $y=1$;
" (1) " $\frac{1}{z}=2$, or $z=\frac{1}{2}$, } *Ans.*

2. Given $\frac{3}{8-x}-\frac{8-x}{3}=\frac{x-11}{12}$

Clearing of fractions,
$$36-256+64x-4x^2=19x-x^2-88$$
Uniting and divid., $x^2-15x=-44$
By 2d method, $x=\frac{15}{2}\pm\sqrt{-44+\frac{225}{4}}$
Reducing, $x=\frac{15}{2}\pm\frac{7}{2}$
$\therefore\ x=11$, or 4, *Ans.*

3. Factoring the given quantities, we have
$$x^2+4x-21=(x+7)(x-3)$$
$$x^2-x-56=(x+7)(x-8)$$
Hence $g.\,c.\,d.=x+7$;
And $l.\,c.\,m.=(x+7)(x-3)(x-8)=x^3-4x^2-53x+168$, } *Ans.*

4. Let x = No. of days B requires;
Then $x + 10$ = " " A "
By condition,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}, \text{ part A and B do in 1 day.}$$

Clearing of frac., $12x + 120 + 12x = x^2 + 10x$
Or $x^2 - 14x = 120$
By 2d method, $x = 7 \pm \sqrt{120+49}$
Reducing, $x = 7 \pm 13$
$\therefore\ x = 20$ days, B's time;
And $x + 10 = 30$ days, A's time, } *Ans.*

5. See Arts. 270, 271.

Powers of y,		y^4+	y^3+	y^2+	$y+$	1
Powers of 3,		1	3	9	27	81
Coefficients,		1	4	6	4	1
$x^4 =$	$(y+3)^4 =$	y^4+	$12y^3+$	$54y^2+$	$108y+$	81
$-x^3 =$	$-(y+3)^3 =$	$-$	y^3-	$9y^2-$	$27y-$	27
$2x^2 =$	$2(y+3)^2 =$			$2y^2+$	$12y+$	18
$-3 =$						-3

$\therefore\ x^4 - x^3 + 2x^2 - 3 = y^4 + 11y^3 + 47y^2 + 93y + 69$, *Ans*